KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

THIRD YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS
SECOND SEMESTER, 2021/2022
(JUNE-SEPTEMBER, 2022)

MATH 313: REAL ANALYSIS II

STREAM: Y3 S2 **TIME: 2 HOURS**

DAY: TUESDAY, 12:00 – 2:00 P.M. **DATE: 20/09/2022**

INSTRUCTIONS

1. ***Do not write anything on this question paper.***
2. ***Answer Question ONE [Compulsory] and any other THREE Questions.***

QUESTION ONE

a) Use Weierstrass M-test to show that $\Sigma x n 2 n \infty n=1$ is uniformly convergent 5mks
b) Define the exponential function and list three characteristics for exponential functions 4mks
c) Use ratio test to show whether $\Sigma n n n!$ is convergent or not 5mks
d) Find the values of x for which the series $\Sigma (2x+1) n n 2$ 4mks
e) Let $f\ (x)\{1,\ x\in[a,b]\cap ⁋ -1,\ x\in[a,b]\cap ⁋ c$, show that $f(x)$ is not Riemann integral 6mks
f) State the Darboux's integrability condition 2mks
g) Evaluate $\int x\ d\ e|x| 1 -1$, where $e|x| = \{e x\ 0\leq x\leq 1\ e -x\ -1\leq x\leq 0$ 4mks

QUESTION TWO

h) Prove that a bounded monotonic function is a function of a bounded variation 4mks
i) Prove that a function of bounded variation is necessarily bounded 4mks
j) Prove that the sum of two functions of bounded variation is also of bounded variation 4mks
k) Prove that the product of two functions of bounded variation is also of bounded variation 4mks
l) Find the range of convergence of $\Sigma x n n!$ 4mks

QUESTION THREE

m) Let $f:[a,b]\rightarrow\mathbb{R}$ be a continuous function and $f(a)\neq f(b)$. Prove that if k is any number between $f(a)$ and $f(b)$, then there exists $c\in(a,b)$ such that $f(c)=k$. 5mks
n) Prove that if $f:[a,b]\rightarrow\mathbb{R}$ is a continuous function then f is Riemann-Stieltjes integrable w.r.t α 5mks
o) Let $f(x)=x2$, compute
i) $U(p,f,\alpha)$ given that $\alpha(x)=2x+1\ and\ p=\{0<13<23<1\}$

i ii) Let $P_n=\{0,\frac{1}{n},\frac{2}{n},\ldots,1\}$ compute $\lim_{n\to\infty}U(P_n,f,\alpha)$

ii iii) From ii) above, state whether or not $f\in R(\alpha)$ on [0,1] 10mks

QUESTION FOUR

a) Suppose $f_n\to f$ uniformly on a set E in a metric space. Let a be a limit of E and suppose that $\lim_{n\to\infty}f_n(x)=\lim_{n\to\infty}A_n$. Prove that A_n converges and $\lim_{x\to a}f(x)=\lim_{n\to\infty}A_n$ 7mks

b) State and prove the Cauchy-criterion for uniform convergence 7mks

c) Find the interval and radius of converges of the series $\Sigma(-1)^n(x-1)^n$ using the root test method 6mks

QUESTION FIVE

d) Show that if ΣZ_n is absolutely convergent, then ΣZ_n is convergent, however the converse is not necessarily true 8mks

e) State and prove the intermediate mean value theorem 6mks

f) Show that if $f(x)=k$ is a constant function on $[a,b]$, then $f\in R[a,b]$ and find its integral 6mks