KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

MAIN CAMPUS

SECOND YEAR EXAMINATIONS FOR THE AWARD OF THE DEGREE OF BACHELOR OF SCIENCE (ECONOMICS AND STATISTICS)

FIRST SEMESTER (2023/ 2024)

(SEPTEMBER - DECEMBER 2023)

BECS / ECON 230: MATHEMATICS FOR ECONOMISTS I

STREAM: Y2S1 TIME: 2 HOURS

DAY: DATE:

INSTRUCTIONS

1. Do not write anything on this question paper.
2. Answer **Question ONE (compulsory**) and any other **TWO** questions.
3. Question ONE carries 30 marks, while every other question contains 20 marks each.

QUESTION ONE (COMPULSORY – 30 MARKS)

a) i) Define the term "Mathematical Economics." (2 marks)
ii) Discuss three (3) disadvantages of using mathematics to model real-life economic problems. (3 marks)

b) The demand and supply functions of a two-commodity model are as follows:

$$Qd_1 = 18 - 3p_1 + p_2 \qquad Qd_2 = 12 + p_1 - 2p_2$$

$$Qs_1 = -2 + 4p_1 \qquad QS_2 = -2 + 3p_2$$

Find the equilibrium prices and quantities associated with the model. (4 marks)

c) Given the 3×3 matrix,

$$\begin{pmatrix} 2 & 1 & 6 \\ 5 & 3 & 4 \\ 8 & 9 & 7 \end{pmatrix}$$

i) Calculate all the cofactors of **A.** (3 marks)

ii) Use Laplace expansion theorem to find the determinant of **A.** (1 mark)

iii) Use i) and ii) to find the inverse of **A.** (2 marks)

d) Assume that a firm can sell as many units of its product as it can manufacture in a month at \$18 each. It has to pay out \$240 fixed costs plus a marginal cost of \$14 for each unit produced. How much does it need to produce to break even? (3 marks)

e) If the utility function of an individual takes the form:

$$U = U(X_1, X_2) = (X_1 + 2)^2 (4X_2 + 3)^3$$

Where U is the utility and X_1, X_2 are the quantities of the two commodities consumed. Find the marginal utility function of each of the two commodities. (4 marks)

f) A firm buys the three inputs K, L and R at prices per unit of \$10, \$5 and \$3 respectively. The marginal product functions of these three inputs are:

$$MP_K = 150 - 4K$$

$$MP_L = 72 - 2L$$

$$MP_R = 34 - R$$

What input combination will maximise output if the firm's budget is fixed at \$285?

(4 marks)

g) Find the limit of the following:

i) $\lim_{x \to 2} \frac{3x^2 + x - 10}{x + 2}$ (2 marks)

ii) $\lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25}$ (2 marks)

QUESTION TWO (20 MARKS)

a) i) Define Static Analysis. (2 marks)

ii) Discuss TWO limitations of Static Analysis. (2 marks)

iii) A firm produces the two goods A and B using inputs K and L. Each unit of A requires 2 units of K plus 6 units of L. Each unit of B requires 3 units of K plus 4 units of L. The amounts of K and L available are 120 and 180 respectively. What output levels of A and B will use up all the available K and L? (3 marks)

b) Use Gauss-Elimination to solve for x, y and z, given the following set of simultaneous equations:

$$12x + 15y + 5z = 158$$

$$4x + 3y + 4z = 50$$

$$5x + 20y + 2z = 148$$

(5 marks)

c) Find the total derivative of quantity demanded of product A, where qd_A is a function of the prices of products A, B, C but each of the latter is, in turn, some functions of time.

$$qd_A = 200 - 5P_A^2 + \frac{1}{2} P_B^{\frac{2}{3}} - P_C^{\frac{1}{2}}$$

$$P_A = 10 + \frac{1}{20} t$$

$$P_B = 26 + \frac{1}{5} t$$

(5 marks)

d) A sum of $ 450, 000 is invested at a monthly interest rate of 0.6%. What will the final sum be after 18 months? What is the corresponding AER? (3 marks)

QUESTION THREE (20 MARKS)

a) Briefly describe the nature of comparative statics. (2 marks)

b) Given the following demand and supply functions, find the equilibrium prices and quantities by using **the inverse matrix method.**

$$qd_A = 150 - 2\,P_A - P_B + P_C$$

$$qd_B = 280 - 3P_A - 5P_B + 4P_C$$

$$qd_C = 140 + P_A + 2P_B - 7P_C$$

$$qs_A = 12P_A - 43$$

$$qs_B = 7P_B - 144$$

$$qs_c = 15P_C - 18$$

(6 marks)

c) Given the following functions:

$$y_1 = 3x_1^2 + x_2$$
$$y_2 = 9x_1^4 + 6x_1^2\,(x_2 + 4\,) + x_2(x_2 + 8) + 12$$

Test the existence of functional dependence. (5 marks)

d) In an economy which engages in foreign trade, it is assumed that:

$$Y = C + I + G + X - M$$
$$C = 0.9\,Y_t$$
$$Y_t = (1 - t)Y \quad and \quad M = 0.15Y_t$$

The usual notation applies, and the following values are given:I = $ 200m, G = 270m, X = $ 180m, and t = 0.2.

i)What is the equilibrium value of Y? (3 marks)
ii) What is the balance of payments surplus / deficit at this value? (1 mark)

e) An initial investment of $ 50, 000 increases to $ 56711.25 after 2 years. What interest rate has been applied? (3 marks)

QUESTION FOUR (20 MARKS)

a) Given the input matrix and the final demand vector:

$$A = \begin{bmatrix} 0.05 & 0.25 & 0.34 \\ 0.33 & 0.10 & 0.12 \\ 0.19 & 0.38 & 0 \end{bmatrix} \qquad d = \begin{bmatrix} 1800 \\ 200 \\ 900 \end{bmatrix}$$

i)Provide the economic meaning of the elements 0.33, 0 and 200. (2 marks)

ii) Give the economic meaning (if any) of the third-row sum. (1 mark)

iii)Write out the specific input-output matrix equations for this model. (2 marks)

iv)By Cramer's rule, find the solution output levels of the three industries in the above model. (5 marks)

b) A proportional sales tax v is imposed in a competitive market where:

$$p_d = 800 - 4d \quad and \quad p_s = 50 + 5q$$

Derive reduced form equations for the equilibrium values of p and q in terms of the tax rate v and use them to predict $p$when v is 15%. (4 marks)

c) A firm faces the demand schedule$q = 40 - p^{0.5}$ (where $p^{0.5} \geq 0,\ q \leq 40$) and the cost schedule $TC = q^3 - 2.5q^2 + 50\,q + 16$. What price should it charge to maximise profit? (3 marks)

d) Find the Marginal Revenue function corresponding to the demand schedule:

$$q = (60 - 2.5p)^{0.5}$$

(3 marks)

ECON 230 / BECS 230: MATHEMATICS FOR ECONOMISTS 1 COURSE OUTLINE

Course title: Mathematics for Economists I

Course code: ECON 230 / BECS 230

Year and semester: Year 2 Semester 1

Pre-requisite: None

Contact hours: 45 **Course lecturer**: Dr. Songa, M.A

Purpose of the course: This course enables the learner to identify and appreciate the relationship between Mathematics and economics and how different areas in Mathematics can be applied to solve Economicsissues.

Objectives of the course:

- The student should be able to use knowledge of Mathematics such as Algebra and set theory to solve systems ofequations.
- The student should be able to perform equilibrium analysis and optimization analysis using various aspects ofMathematics.

Course content: The nature of Mathematical Economics. Mathematical Models. Elementary Algebra. Binomial Expansion. Real numbers. Set theory. Relations and functions. Linear and non-linear functions. Equations and inequalities. Simultaneous equations. Static equilibrium analysis. Matrix Algebra. Input-Output analysis. The nature of comparative Statics. Limits and continuity.

Course Outline

Week	Course Content	Topics
Week 1	The nature of Mathematical Economics	• Mathematical versus non-mathematicaleconomics • Mathematical economics versus econometrics
Week 2	Mathematical Models	• The partial marketmodel • General marketmodel • National incomemodel • Keynesianmodels
Week 3	Elementary Algebra	• Types of functions (Constant, polynomial, rational,irrational) • Algebraic functions • Non-algebraicfunctions
Week 4	Binomial Expansion, Real numbers, Set Theory,Relations and Functions	• Binomialexpansion • Venn diagrams and applications of set theory • The real numbersystem
Week 5	Simultaneous Equations	

Week 5	Static Equilibrium Analysis	• The meaning ofequilibrium • Partial Market Equilibrium (Linear and non-linearmodels)
Week 6		• General market equilibrium • Equilibrium in national income model

Week 7	Matrix Algebra	• Matrixoperations • Identity/null matrices • Transposes ofmatrices
Week 8		• Conditions fornon-singularity • Test ofnon-singularity • Inverses using Gauss-Jordan (Consider 3 by 3matrices)
Week 9	Input-Output Analysis	• Applications of matrices to Market and National Incomemodels • Leontif Input-outputmodels • Limitations of StaticAnalysis
Week 10	The nature of comparative statistics	• Rules ofdifferentiation • Partial differentiation and its applications • Jacobiandeterminants
Week 11		• Differentials/ Totaldifferentials • Totalderivatives • Derivatives of Implicitfunctions • Comparative statics of the general functionmodels • Limitations of comparative statistics
Wek 12	Limits and Continuity	• The concept oflimits • Limittheorems • Continuity and differentiability of afunction

Mode of delivery: Lectures, Assignments, Group-work

Instructional materials: Text books, handouts, online notes.

Course assessment: Continuous Assessment Test – 30%, Examination – 70%, Total – 100%**(CAT 1 at Week 5, CAT II at Week 9)**

Core reading materials:

1. M. S. Mukras, Elements of Mathematical Economics, Kenya Literature Bureau,2004.
2. F. S. Budnick, Applied Mathematics for Business, Economics, and the Social Sciences, McGraw-Hill, Inc.,1993.

Recommended reference materials:

1. L. C. Peck, Basic Mathematics for Management and Economics, Scott Foresman and Company, 1970.
2. C. Dinwiddy, Elementary Mathematics for Economists, Oxford University Press, 1967.