KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

MAIN CAMPUS

THIRD YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF BACHELOR OF ECONOMICS AND STATISTICS

[DECEMBER , 2023]

MATH 446: SAMPLING METHODS II

STREAM: ECON Y4S1 **TIME: 2 HOURS**

DAY: **DATE:**

INSTRUCTIONS

1. Do not write anything on this question paper.
2. Answer questions ONE and any other TWO questions.
3. Question one carries 30 marks and other questions carry 20 marks each.

QUESTION ONE (COMPULSORY 30 MARKS)

(a) Briefly distinguish between double sampling plan and two-stage cluster sampling. (3 marks)

(b) If $\overline{y}_{sy}$ is the systematic sample mean, show that it is unbiased for the population Mean $\overline{Y}$. (4 marks)

(c) Consider a population consisting 6 units with the following values of a variable Y, 5, 4, 8, 2, 3 and 14. A random sample of size 3 using` a systematic procedure is required from this population. Write down all the possible random samples and verify that the sample mean is unbiased for the population mean. (5 marks)

(d) Using the usual notations, show that systematic sampling is more precise than simple random sampling without replacement if $e < \frac{-1}{N-1}$ where e is the intra-class correlation coefficient.

(4 marks)

(e) If the lot fraction defective p = 0.01, n = 89 and C =2. Find the probability of acceptance P_a.

(4 marks)

(f)The following data is a simple random sample without replacement from a population of size 100: 5, 8, 4, 2, 6. Pointing out the assumptions you make, calculate the 95% confidence interval for the population mean $\overline{Y}$. (5 marks)

(g)A population of size 100 is divided into strata A and B with stratum A having 40 units. An initial simple random sample of size 4 is taken from each of the two strata obtaining the following values:

A:	12	13	15	16
B:	0	12	24	16

It is desired to take a simple random sample of size 30 to estimate the population parameter. Allocate the sample size to the two strata using optimum allocation scheme.

(5 marks)

QUESTION TWO (20 MARKS)

Suppose that the population consist of a linear trend of the form y_i= i for I = 1, 2,- - - -, N. Show that:

(i) $S^2 = \frac{N(N+1)}{12}$ (ii) Var $\overline{y}_{ran} = \frac{(K-1)(N-1)}{12}$ (iii) Var $\overline{y}_{st} = \frac{K^2-1}{12n}$ (iv) Var $\overline{y}_{sy} = \frac{K^2-1}{12}$

Hence deduce that Var $\overline{y}_{st} \leq$ Var $\overline{y}_{sy} \leq$ Var $\overline{y}_{ran}$ (20 marks)

QUESTION THREE (20 MARKS)

(a) Consider a population consisting 12 units whose values on some characteristics are: $\underline{Y}$ = 2, 3, 8, 14, 10, 32, 18, 15, 16, 4, 5, and 7. A systematic sample of size 4 is required and the objective is to estimate the population mean.

(i) Estimate the population mean using a sample that contains unit number 4.

(ii) Calculate the variance of the estimate obtained in (i) above (12 marks)

(b) Suppose we wish to find a sequential sampling plan with $p_1 = 0.02$, $p_2 = 0.09$, $\beta = 0.01$ and $\alpha = 0.05\%$. Find the acceptance line and rejection line. (8 marks)

QUESTION FOUR (20 MARKS)

(a) A double sampling plan has n_1= 50, C_1= 1, n_2= 100 and C_2 = 3. If the lot fraction defective p = 0.05. Find P_a (Probability of acceptance) (9 marks)

(b) A manufacture of band saws wants to estimate the average repair cost per month for the saws he has sold to certain industries. He cannot obtain a repair cost for each saw, but he can obtain the total amount spent for saw repair and the number of saws owned by each industry. Thus he decides to use cluster sampling with each industry as a cluster. The manufacturer selects a simple random sample of size

n = 20 from the N = 96 industries he serves. Estimate the average repair cost per saw for the past month and place a bound on the error of estimation, given that:

m_i is the number of saws

y_i is the total repair cost for the past month

$\sum_{i=1}^{20} m_i$ = 130, $\sum_{i=1}^{20} m_i^2$ = 1118, $\sum_{i=1}^{20} y_i$ = 2565, $\sum_{i=1}^{20} y_i^2$ = 460225 and $\sum_{i=1}^{20} m_i y_i$ = 22285 (11 marks)