

KISII UNIVERSITY
UNIVERSITY EXAMINATIONS

THIRD YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF
BACHELOR OF ECONOMICS AND STATISTICS

FIRST SEMESTER 2023/2024

[DECEMBER, 2023]

MATH 452: TESTS OF HYPOTHESIS

STREAM: ECON Y3S2

TIME: 2 HOURS

DAY:.....

DATE.....

INSTRUCTIONS

1. Do not write anything on this question paper.
2. Answer question ONE and any other TWO questions.

QUESTION ONE (COMPULSORY 30MARKS)

(a) Briefly distinguish between the following terms.

- i. The power function and hypothesis test.
- ii. Type I error and type II error.
- iii. Critical region of a test and the best critical region of a test (BCR).

(6 marks)

(b) Let $X \sim B(4, p = \theta)$. To test the hypothesis

$$H_0 : \theta = \frac{1}{2} \text{ against}$$

$$H_1 : \theta = \frac{5}{6} \quad \text{The significance level } \alpha \text{ is set at } \alpha = \frac{1}{16}. \text{ Obtain a BCR for testing } H_0 \text{ vs } H_1$$

[6marks]

(c) Let $X \sim P(\lambda)$. Consider the simple hypothesis $H_0: \lambda = \frac{1}{2}$ against the alternative $H_1: \lambda \leq \frac{1}{2}$ such that $\Omega = \{\lambda: \lambda \leq \frac{1}{2}\}$. If x_1, x_2, \dots, x_{12} denote a random sample of size 12 from this distribution, we reject H_0 if the observed value of $y = x_1 + x_2 + \dots + x_{12} \leq 2$. If $\beta[\lambda]$ is the power function of the test find the power at $\beta[1/2], \beta[1/3], \beta[1/4], \beta[1/6]$ and $\beta[1/12]$ and sketch the graph of $\beta[\lambda]$. What is the significance level of the test? (6 marks)

(d). Suppose $X \sim N[\mu, \delta^2]$ where δ^2 is known. By taking a random sample of size n construct a best test [M.P] for testing

$H_0: \mu = \mu_0$ against

$H_1: \mu > \mu_1$ (6marks)

(e). Describe the likelihood ratio test procedure. (6 marks)

QUESTION TWO { 20 MARKKS }.

a. State and prove Neyman – Pearson theory. (12 marks)

b. Let X_1, X_2, \dots, X_{10} be a random sample of size 10 from a normal distribution, $N[0, \delta^2]$. Find the BCR of size $\alpha = 0.05$ for testing

$H_0: \delta^2 = 1$ against

$H_1: \delta^2 = 2$ (8 marks)

QUESTION THREE (20 MARKS)

(a) Consider a simple regression model

$Y_i = \alpha + \beta x_i + e_i$ where α, β are constants and $e_i \sim N(0, \delta^2)$. Derive a test statistic for testing the hypothesis $H_0: \beta = 0$ against $H_1: \beta \neq 0$. (15 marks)

(b) Consider the following data

X	-2	-1	0	1	2
Y	0	0	1	1	3

Fit a straight line and test $H_0: \beta=0$ against $H_1: \beta \neq 0$ at 5% level of significance. (5 marks)

QUESTION FOUR (20 MARKS)

Let X_1, X_2, \dots, X_n be a random sample of size n from a normal distribution

$X \sim N(\theta_1, \theta_2)$. Use the like hood ratio test to obtain the BCR of size α for testing

$H_0: \theta_1 = 0$ against $H_1: \theta_1 \neq 0$. (20marks)

