a. Define the following terminologies as used in mathematics for computing:

    i.   Subset`   [3 marks]

    ii.   Universal Set   [3 marks]

    iii.   Venn diagram   [3 marks]

b. The quality controller in a given firm had an accurate record of all the iron bars produced in May 2017. The following data shows those records

Bar lengths(cm)	201-250	251-300	301-350	351-400	401-450	451-500	501-550
No. of bars	25	36	49	80	51	42	30

    i.   Calculate the mean of the length of bars   [6 marks]

    ii.   Calculate the standard deviation of the of the lengths of the bars   [6 marks]

    iii.   Find the median   [4 marks]

Question two [15 marks]

a. Perform the following arithmetic operations (show your calculations)

    i. $111011 + 111001$ [3 marks]

    ii. $10001.011 + 101010$ [3 marks]

    iii. $110101 - 11011$ [3 marks]

    iv. $11010101.10111 - 11.01010$ [3 marks]

b. The frequency distribution for the value of resistance in ohms of 48 resistors is as shown. Determine the mean value of resistance.   [5 Marks]

Range	Frequency
20.5–20.9	3
21.0–21.4	10
21.5–21.9	11
22.0–22.4	13
22.5–22.9	9
23.0–23.4	2

Question Three [15 marks]

a. Determine the standard deviation from the mean of the set of numbers: {5, 6, 8, 4, 10, 3} correct to 4 significant figures. [5 Marks]

b. Use the laws and rules of Boolean algebra to simplify the following expressions:

    i. $(P+\overline{P}\cdot Q)\cdot(Q+\overline{Q}\cdot P)$ [5 marks]

    ii. $F\cdot G\cdot\overline{H}+F\cdot G\cdot H+\overline{F}\cdot G\cdot H$ [5 marks]

Question four [15 marks]

a. Differentiate between the following terms as used in set theory

    i.   Union and Intersection [4 marks]

    ii.   Universal Set and Empty Set [4 marks]

b. Determine the mean, median and mode for the set:

    {2, 3, 7, 5, 5, 13, 1, 7, 4, 8, 3, 4, 3} [4 marks]

c. What are the differences between Straight Line, Quadratic Graphs and Cubic Equations [3 marks]

Question five [15 marks]

a. Convert 3714_{10} to a binary number, via octal. [6 marks]

b. Convert

    i. $5F_{16}$ into its decimal equivalent [3 marks]

    ii. 132_{10} into its hexadecimal equivalent [3 marks]

c. Convert 47.40625_{10} to a binary number. [3 marks]

Question six [15 marks]

a. Let $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6, 7\}$, $C = \{2, 3, 8, 9\}$. Then which elements will be contained in the following:

    i. $A \cup B$ [2 Marks]
    ii. $A \cup C$ [2 Marks]
    iii. $B \cup C$ [2 Marks]
    iv. $A \cap B$ [2 Marks]
    v. $A \cap C$ [2 Marks]
    vi. $B \cap C$ [2 Marks]

MARKING SCHEME

Question One

a.

    i.   Subset   - Suppose every element in a set *A* is also an element of a set *B*, that is, suppose $a \in A$ implies $a \in B$. Then *A* is called a *subset* of *B*.

    ii.   Universal set- All sets under investigation in any application of set theory are assumed to belong to some fixed large set called the *universal set*

    iii.   Venn diagram is a pictorial representation of sets in which sets are represented by enclosed areas in the plane. The universal set **U** is represented by the interior of a rectangle, and the other sets are represented by disks lying within the rectangle.

b.

Bar lengths (cm)	No. of bars(f)	Class mid point (x)	fx	fx²
201 – 250	25	225.5	5637.5	1271256.25
251 – 300	36	275.5	9918	2732409
301 – 350	49	325.5	15949.5	5191562.25
351 – 400	80	375.5	30040	11280020
401 – 450	51	425.5	21700.5	9233562.75
451 – 500	42	475.5	19971	9496210.50
501 - 550	30	525.5	15765	8284507.50
	313		118981.50	47489526

Calculate the standard deviation of the lengths of the bars

$$\therefore \text{ standard deviation, } \sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$= \sqrt{\frac{47489526}{313} - \left(\frac{118981.50}{313}\right)^2}$$

$$= 84.99 \text{ cm}$$

Question two

a.

    i. $11010 + 101011 = 1000101$

    ii. $10111.011 + 11010 = 110001.011$

iii. $110101 - 11011 = 11010$

iv. $11010101.10111 - 11.01010 = 110010.01101$

b.

For grouped data, the mean value is given by:

$$\bar{x} = \frac{\sum (f\,x)}{\sum f}$$

where f is the class frequency and x is the class mid-point value. Hence mean value,

$$\bar{x} = \frac{\begin{array}{c}(3 \times 20.7) + (10 \times 21.2) + (11 \times 21.7)\\ + (13 \times 22.2) + (9 \times 22.7) + (2 \times 23.2)\end{array}}{48}$$

$$= \frac{1052.1}{48} = 21.919.$$

Question three

a.

The arithmetic mean,

$$\bar{x} = \frac{\sum x}{n} = \frac{5 + 6 + 8 + 4 + 10 + 3}{6} = 6$$

Standard deviation, $\sigma = \sqrt{\left\{\dfrac{\sum (x - \bar{x})^2}{n}\right\}}$

The $(x - \bar{x})^2$ values are: $(5 - 6)^2$, $(6 - 6)^2$, $(8 - 6)^2$, $(4 - 6)^2$, $(10 - 6)^2$ and $(3 - 6)^2$.

The sum of the $(x - \bar{x})^2$ values,

i.e. $\sum (x - \bar{x})^2 = 1 + 0 + 4 + 4 + 16 + 9 = 34$

and $\dfrac{\sum (x - \bar{x})^2}{n} = \dfrac{34}{6} = 5.\dot{6}$

Hence, standard deviation,

$$\sigma = \sqrt{\left\{\frac{\sum (x - \bar{x})^2}{n}\right\}} = \sqrt{5.6}$$

$$= 2.380, \text{ correct to 4 significant figures}$$

b.

i.

$$= P \cdot (Q + \overline{Q} \cdot P)$$
$$+ \overline{P} \cdot Q \cdot (Q + \overline{Q} \cdot P)$$
$$= P \cdot Q + P \cdot \overline{Q} \cdot P + \overline{P} \cdot Q \cdot Q$$
$$+ \overline{P} \cdot Q \cdot \overline{Q} \cdot P$$
$$= P \cdot Q + P \cdot \overline{Q} + \overline{P} \cdot Q$$
$$+ \overline{P} \cdot Q \cdot \overline{Q} \cdot P$$
$$= P \cdot Q + P \cdot \overline{Q} + \overline{P} \cdot Q + 0$$
$$= P \cdot Q + P \cdot \overline{Q} + \overline{P} \cdot Q$$
$$= P \cdot (Q + \overline{Q}) + \overline{P} \cdot Q$$
$$= P \cdot 1 + \overline{P} \cdot Q$$
$$= P + \overline{P} \cdot Q$$

ii.

$$= F \cdot G \cdot (\overline{H} + H) + \overline{F} \cdot G \cdot H$$
$$= F \cdot G \cdot 1 + \overline{F} \cdot G \cdot H$$
$$= F \cdot G + \overline{F} \cdot G \cdot H$$
$$= G \cdot (F + \overline{F} \cdot H)$$

Question four

a.

    i.   The union of two sets A and B, denoted by A ∪ B, is the set of all elements which belong to A or to B; that is, A ∪ B = {x | x ∈ A or x ∈ B} The intersection of two sets A and B, denoted by A ∩ B, is the set of elements which belong to both A andB; that is, A ∩ B = {x | x ∈ A and x ∈ B}

    ii.   All sets under investigation in any application of set theory are assumed to belong to some fixed large set called the *universal set* which we denote by **U** unless otherwise stated or implied. Given a universal set **U** and a property P, there may not be any elements of **U** which have property P. For example, the following set has no elements:

a. Straight Line - The general equation of a straight line is y=mx +c, where m is the gradient i.e. dy/dx and c is the y-axis intercept., Quadratic Graphs - The general equation of a quadratic graph is y=ax2 +bx +c, and its shape is that of a parabola. and Cubic Equations - The general equation of a cubic graph is y=ax3 + bx2 +cx +d.

Question five [15 marks]

```
8|3714    Remainder
8|_464    2
8|_58     0
8|_7      2
   0      7
          7  2  0  2
```
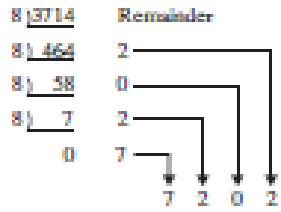
$7202_8 = 111\ 010\ 000\ 010_2$

c.

      i. $5F_{16}$

        $5 \times 16^1 + 15 \times 16^0$

        $90 + 15$

        $= 105_{10}$

     ii. 132_{10}

132/16	8	4
8/16	0	8

        $= 84_{16}$

d. Convert 47.40625_{10} to a binary number

		Remainder
47/2	23	1
23/2	11	1
11/2	5	1
5/2	2	1
2/2	1	0
½	0	1

101111_2

0.40625	x	2	=	0.8125
0.8125	x	2	=	1.625
0.625	x	2	=	1.25
0.25	x	2	=	0.5
0.5	x	2	=	1.0

101111.01101_2