KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS
SECOND SEMESTER 2022/2023
[MAY, 2023]

MATH 841: NUMERICAL ANALYSIS II

STREAM: Y1 S2 **TIME: 3 HOURS**

DAY: TUESDAY, 9:00 – 12:00 P.M. **DATE: 02/05/2023**

INSTRUCTIONS

1. ***Do not write anything on this question paper.***
2. ***Answer Question ONE and any other TWO (2) Questions***
3. ***Show all the relevant working.***

QUESTION ONE *compulsory* (30MKS)

a. Solve the linear system using Cholesky method

$$\begin{bmatrix} 4 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 \\ 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

[7marks]

b. Find the conditional number of the system given as $\begin{bmatrix} 2.1 & 1.8 \\ 6.2 & 5.3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2.1 \\ 6.2 \end{bmatrix}$, what is the condition of the system [4marks]

c. Show that the Jacobi iterative method can be written as $x^{(k)} = Hx^{(k-1)} + C$, where $H = -D^{-1}(L+U)$ and $C = D^{-1}b$. [4marks]

d. Solve the linear system using partial pivoting

$$\begin{aligned} x_1 + x_2 + 3x_4 &= 4 \\ 2x_1 + 2x_2 - x_3 + x_4 &= 1 \\ 3x_1 - x_2 - x_3 + 2x_4 &= -3 \\ -x_1 + 2x_2 + 3x_3 - x_4 &= 4 \end{aligned}$$

(5marks)

e. Consider the linear system $Ax = b$ given by

$$4x_1 + x_2 + 2x_3 = 4$$
$$3x_1 + 5x_2 + x_3 = 7$$
$$x_1 + x_2 + 3x_3 = 3$$

i. Set up the SOR iterative scheme for the solution

ii. Find the optimal relation factor and hence find the rate of convergence of the scheme

iii. Using the optimal relation factor iterate three times starting with zero initial vector. [10marks]

QUESTION TWO (15MKS)

a. Solve the linear programming problem using the systematic trial error method;

$$\max z = 3x_1 + 2x_2$$

Subject to the constraints

$$x_1 + 2x_2 \leq 430$$
$$3x_1 + x_2 \leq 460$$
$$x_1 \geq 0,\ x_2 \geq 0$$

[5marks]

b. Use the Simplex method to find the optimum of the linear programming problem;

$$\max z = 4x_1 + 10x_2$$

subject to the constraints

$$2x_1 + x_2 \leq 50$$
$$2x_1 + 5x_2 \leq 100$$
$$2x_1 + 3x_2 \leq 90$$
$$x_1 \geq 0,\ x_2 \geq 0$$

[5marks]

c. A firm produces three types of canvas. Type 1, type 2, and type 3. Three kinds of material A, B, C are required. One-unit length of 1^{st} canvas require 2 meters of A, 3 meters of C. One-unit length of 2^{nd} canvas require 3 meters of A, 2 meters of B and 3 meters of C. One-unit length of 3^{rd} canvas require 5 meters of B, 4 meters of C. the company has a stock 8 meters of A, 10 meters of B and 15 meters of C. the cost of making a unit length of type 1, type 2 and type 3 are kshs. 30, 50, and 40 respectively. Formulate a linear programming problem and solve it. [5marks]

QUESTION THREE (15MKS)

For the solution of the system of equation

$$4x_1 + 3x_2 = 24$$
$$3x_1 + 4x_2 - x_3 = 30$$
$$-x_2 + 4x_3 = -24$$

a. Set up the Gauss-seidel and SOR iterative scheme for the solution and iterate three times starting with $x^{(0)} = 0$ and $w = 1.25$ [12marks]

b. Find the optimal relaxation factor for the SOR method. [3marks]

QUESTION FOUR (15MKS)

a. Show that the iterative matrix for SOR method is $x^{(k)} = Hx^{(k-1)} + C$ where $H = (D - WL)^{-1}\{(1-w)D + wu\}$ and $C = w(D - WL)^{-1}b$,D,L,U and w hence usual meaning [7 marks]

b. Prove that the infinite series $I + A + A^2 + A^3 + \ldots$ converges if $\lim_{m\to\infty} A^m = 0$ and the converged series is $(I - A)^{-1}$. [4mks]

c. Find the inverse of a matrix If A given as, $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ using the iterative method given that $B = \begin{bmatrix} 1.8 & -0.9 \\ -0.9 & 0.9 \end{bmatrix}$. [4mks]

QUESTION FIVE (15MKS)

a. Consider the non-linear system

$$x_1^2 - 10x_1 + x_2^2 + 8 = 0$$

$$x_1 x_2^2 + x_1 + 10x_2 + 8 = 0$$

Transform the non-linear system into the fixed point problem $x_1 = g_1(x_1, x_2)$ and $x_2 = g_2(x_1, x_2)$ [5marks]

b. Show that no eigenvalue of a matrix A exceeds the norm of a matrix i.e $P(A) \le \|A\|$ [5marks]

c. The linear system $\begin{bmatrix} 1 & 2 \\ 1.0001 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ has solution (1,1). Change A slightly to $\begin{bmatrix} 1 & 2 \\ 0.9999 & 2 \end{bmatrix}$ and consider system $\begin{bmatrix} 1 & 2 \\ 0.9999 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3.00001 \end{bmatrix}$. Compute the new solution using the five digit arithmetic. Estimate K(A) [5marks]