Math 111: BASIC CALCULUS 1 **TIME: 2 HOURS**

INSTRUCTIONS

1. **Do not write on this question paper**
2. **Answer questions ONE and any other three questions**
3. **Show all the relevant workings**

QUESTION ONE (25MKS)

a) Differentiate the following equation

i) $y= x^{-2}(x-x^3)$ (3mks)

ii) $y=x(5x-2) + 3(x + 2)^2$ (3mks)

(b) Use the mid-ordinate rule to find the area enclosed by $y= -x^2 + 2x + 8$, the y axis and the x axis using four strips (4mks)

b) Integrate the following

i) $\int_2^4 (x^3 - 3x^2 + 2x - 1)dx$ (3mks)

c) Estimate the area enclosed by the curve $y=\frac{1}{2}x^2 + 1$ between x= 1 and x= 4 using mid-ordinate rule (4mks)

d) Find the exact area by integration enclosed by the curve $y = 3x^2 + 2x + 1$ the x-axis x=1 and x=3 (3mks)

QUESTION TWO

a) Integrate the following:

$\int_1^2 (x^2 - 6x + 2)dx$ (3mks)

a) Evaluate the following integrals

i) $\int_0^5 (x^3 + 6x^2 - 4x + 1)\, dx$ (3mks)

ii) $\int_1^4 (4t^3 - t^2 + 2t - 1)\, dt$ (3mks)

b) Find the exact area above and below the x axis hence the total area enclosed by the curve $y= x^2 – 10x + 9$, the x axis and the lines x=4 and x=10 (5mks)

QUESTION THREE

a) Find the exact area by integration enclosed by the curve $y = \frac{x^3 - 3x^2 + 2x}{x}$ the x-axis, x=1 and x=4. (3mks)

b) A particle moves in a straight line such that after its displacement, s metres from a given point is $s= t^4 + 3t^2 + 4$, where t is the time in seconds. Determine the following, when t=4seconds

i) Displacement (2mks)
ii) Velocity (2mks)
iii) Acceleration (2mks)

c) Differentiate the following:

i) $y = \frac{x(x^2 - 1)}{x + 1}$ (3mks)

ii) $\sqrt{x}\,(x^{2/3} - x^{1/2})$ (3mks)

QUESTION FOUR

a) A ball is kicked vertically upwards from a point 0.5m above the ground at a velocity of 16m/s. assuming that acceleration due to gravity is 10m/s, determine

i) An expression for its velocity t seconds later (3mks)
ii) The maximum height reached by the ball. (3mks)

(b) A particle moves along a straight line with a constant acceleration. At t=0, its velocity is u and it is at a fixed point O. if v is the velocity after t seconds. Show that

i) $v = u + at$ (3mks)
ii) $s= ut + \frac{1}{2}at^2$ (3mks)

(c) Estimate the area enclosed by the curve $y=\frac{1}{2}x^2 + 2$, the x-axis, x= 1 and x= 3 using

i) Mid-ordinate rule (3mks)
ii) Integration method (3mks)
iii) Percentage error (3mks)

QUESTION FIVE

a) Find the equation of the tangent to the given curve at the indicated point.
$y=x^3 + 6x^2 – 3x + 1$, (x=0) (5mks)

b) Integrate the following

i) $\int_0^5 x(x^2 – 6x + 5)dx$ (3mks)

ii) $y= x^4 + 3x^2 + 2x$ (3mks)

c) Differentiate the following expression.

$y = \frac{(x^2 + x)(x^{-2} + 1)}{x^2}$ (3mks)

QUESTION SIX

a) Given the curve $y= x^2 – 3x – 1$ find the equation at the point (1, -3) of the;

i) The tangent (4mks)

ii) The normal through (2, 4) (3mks)

b) Find the stationary points on the curve $y= 4x^3 + 9x^2 – 30x + 10$. And state the nature of each point (5mks)

c) Estimate the area bounded by the curve $y=½x^2 + 5$, the x-axis, the line x=1 and x=5 using trapezium rule (4mks)