

KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

MATH 211: CALCULUS II

STREAM: BED SCIENCE Y2S1

TIME: 2 HOURS

DAY:

DATE:

INSTRUCTIONS

1. Do not write anything on this question paper.
2. This paper consist of 3 printed pages.
2. Answer Question **ONE**(**COMPULSORY**) and any other **TWO** questions.

QUESTION ONE (30 MARKS)

a. Evaluate the following integrals;

i) $\int \frac{3x}{(4x^2-1)^5} dx$ (5mks)

ii) $\int \frac{2e^t}{\sqrt{e^t+4}} dt$ (5mks)

iii) $\int_2^3 \frac{x^3-2x^2-4x-4}{x^2+x-2} dx$ (5mks)

b. Show that $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1} \frac{x}{a}$. Hence find $\int_0^4 \frac{1}{\sqrt{16-x^2}} dx$ (5mks)

c. Evaluate the value of $\int_1^2 (3x^2 e^{\frac{1}{2}}) dx$ (5mks)

d. The displacement S metres of a particle at time t sec is given by $S = 2t^3 - 5t^2 + 4t - 3$. Show that the average velocity is attained twice on the interval $t = 0$ sec and $t = 2$ sec. Find also average velocity reached. (5mks)

QUESTION TWO (20MARKS)

- a. Given that $z = x \cos y - y \cos x$. Show that $\frac{\delta^2 z}{\delta x^2} + \frac{\delta^2 z}{\delta y^2} + 2 = 0$ (4mks)
- b. Given that $z = \sin^2 \theta \cos 3\theta$. Show that $\frac{\delta^2 z}{\delta \theta^2} + \frac{\delta^2 z}{\delta \theta^2} + 13z = 0$ (4mks)
- c. The pressure, p and volume, v of a gas are related by $p v^{1.2} = c$. Find the approximate percentage change in c when pressure increases by 2.2% and volume decreases by 0.86%. (6mks)
- d. If $z = f(x, y)$ such that $z = e^{\frac{y}{2}} \ln(2x+3y)$. Find rate of change in z when $x = 1$ cm and $y = 2$ cm given that x is increasing at 5cm/s and y is decreasing at 4cm/s. (6mks)

QUESTION THREE (20MKS)

- a. Evaluate $\int 3 \sec^2 x \tan x \, dx$. Hence find $\int_0^{\frac{\pi}{12}} \sec^2 x \tan x \, dx$ (6mks)
- b. Determine the integrals in each case;
- i) $\int \frac{x}{x^2+a^2} \, dx$ (4mks)
- ii) $\int \sin^2 t \cos^2 t \, dt$ (4mks)
- iii) $\int \frac{3+6x+4x^2-2x^3}{x^2(x^2+3)} \, dx$ (6mks)

QUESTION FOUR (20MKS)

- a. State the Rolle's theorem. (2mks)
- b. Find the value of x for which Rolle's theorem is satisfied for the function $f(x) = \sqrt{x-1}$ for the interval $x = 1$ and $x = 3$ (6mks)
- c. Use Rolle's theorem to show that $x^5 + 4x = 1$ has exactly one solution. (4mks)

d. Test whether the function $f(x) = x^{2/3}$ satisfy the mean value theorem for the interval $x = -8$ and $x = 8$. (2mks)

e. Show that $\int_{\sqrt{3}/2}^{3/2} \frac{1}{3+4x^2} dx = \frac{\pi\sqrt{3}}{72}$ (6mks)

QUESTION FIVE (20MKS)

a. State the Taylors theorem (2mks)

b. Use Taylors theorem to expand $\sin(\frac{\pi}{6} + h)$ in ascending powers of h as far as the term in h^4 . Hence estimate the value of $\sin(\frac{\pi}{6} + h)$ given that $\sin\pi/6 = 0.5$ (6mks)

c. Use Taylor's theorem to approximate the value of $\frac{1}{(2.03)^3}$ to 3 d.p. (6mks)

d. Expand $e^{(2a+e)}$ upto e^2 by using Taylors theorem. Hence estimate $e^{3.072}$ correct to 2 d.p. given that $e^3 = 20.086$. (6mks)

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