MATH 420: PARTIAL DIFFERENTIAL EQUATIONS

INSTRUCTIONS: ANSWER QUESTION ONE [COMPULSORY] AND ANY OTHER THREE.

QUESTION ONE [25 MARKS]

a)(i)Explain the term partial differential equation(1mark)

(ii) Differentiate between order and degree of a differential equation. (2 marks)

b) (i) Form a partial differential equation by eliminating arbitrary constants from the equation

Z = 2ax + by where a and b are the constants.(3marks)

(ii) Form a partial differential equation by eliminating arbitrary function f from the equation

$f(x^2 + y^2 + z^2), (x + y + z) = 0$ (5 marks)

c) (i) Define a curve as used in partial differential equation.(1mark)

(ii) The equation of a sphere centre (0,0,0) is given by $2x^2 + 3y^2 + z^2 = a^2$, determine the direction ratios normal to the surface of the sphere. (4 marks)

(iii) Find the integral curves of the equation below by any method of grouping. (5 marks)

$$\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$$

d) Solve the following Pfaffian differential equation by separation of variables.

zdx + zdy +(x+y+sinz)dz = 0 (4 marks)

QUESTION TWO [15 MARKS]

(a) Show that the direction cosines of the tangent at the point (x,y,z) to the conic $ax^2+by^2+cz^2 = 1$, $x+y+z = 1$ are proportional to ((by-cz), (cz-ax), (ax-by)). (6 marks)

(b)Given the equation below, $\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}$ is of the form $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$, find the integral curves by choosing values of P^l, Q^l and R^l such that $P^ldx + Q^ldy + R^ldz = 0$ is exact. (9 marks).

QUESTION THREE [15 MARKS]

(a) Define Orthogonal trajectories(2marks)

(b) Find the integral curves of the equation below by any method of grouping. (5 marks)

$$\frac{dx}{xy} = \frac{dy}{y} = \frac{dz}{yz(x+1)}$$

(c) Find the orthogonal trajectories on the conic $x^2 + y^2 = z^2 \tan^2 \alpha$ of its intersection with the family of planes parallel to z = c. (8 marks)

QUESTION FOUR [15 MARKS]

a)Given a Pfaffian differential equation $xz^3dx – zdy + 2ydz = 0$

(i)Test if it is integrable.(4 marks)

(ii)Find its solution.(5 marks)

b) Given an Homogenous equation $(yz + z^2)dx – xzdy + x(z + y)dz = 0$, solve by using substitutions x = uz and y = vz where u=u(x,y) and v=v(x,y). (6 marks)

QUESTION FIVE [15 MARKS]

(a) An equation of the form $P\frac{\partial z}{\partial x} + Q\frac{\partial z}{\partial y} = R$ is a general Lagrange's equation. Explain the meaning of a semi-linear lagrange's equation.(2marks)

(b) Solve Lagrange's equation $3\frac{\partial z}{\partial x} + 2\frac{\partial z}{\partial y} = 1$ by the method of grouping and find its complete solution.(5 marks)

(c) Use the method of characteristics to solve the Lagrange's equation:

$U_t + xU_x = 0$ where $U(x,0) = g(x)$. (8 marks)

QUESTION SIX [15 MARKS]

(a)Show that the equations $xp = yq$, $z(xp + yq) = 2xy$ are compatible and hence solve them.(7 marks)

(b) Use Charpit's method to find the complete solution of the non-linear p.d.e

$q = -xp+p^2$ (8 marks)