# KISII UNIVERSITY

## UNIVERSITY EXAMINATIONS

### SPECIAL EXAMINATION

**FIRST YEAR EXAMINATION FOR THE AWARD OF**
**DEGREE IN BACHELOR OF SCIENCE IN MATHEMATICZ & COMPUTING**
**FIRST SEMESTER 2021/2022**
**(JULY, 2022)**

**MATH 111: CALCULUS 1**

**STREAM: Y1 S1** **TIME: 2 HOURS**

**DAY: WEDNESDAY, 8.00 AM – 10.00 AM** **DATE: 20/07/2022**

**INSTRUCTIONS:**

***1. Do not write anything on this question paper.***
***2. Answer Question ONE (Compulsory) and any other TWO Questions.***

**QUESTION ONE (30 marks)**

a) If $x = \cos tandy = 1 - \sin 2t$,find $\frac{dy}{dx}$ $and$ $\frac{d^2y}{dx^2}$ (4marks)

b) State thre conditions for a function to be continuous at a point $x = a$ (3marks)

c) Find $\frac{dy}{dx}$ if $y = e^{3x}(\sin 2x)$ (3marks)

d) Find $f^{/}(x)$ for$f(x) = x^2 + 4x - 7$from first principle (4marks)

e)

f) Investigate the nature of the turning point of the curve $y = 3x^2+6x^2 - 15x + 51$

i) Locate the turning point (5marks)

ii) Determine the nature of the turning point

g) Find the equation of the tangent and the normal to the curve $y = x^3 - 2x^2 + 3x - 1$ at the point $P(2, 5)$. (6marks)

h) Evaluate the limit $\lim_{x \to 0} \frac{\sin(3x)}{\sin(8x)}$ (5marks)

**QUESTION TWO (20 marks)**

a) Find $f^{'}(x)$ given that $f(x) = \log_3 \sin(2x^2 + 5)$

(5marks)

b) A 15m ladder is placed to rest against the wall so that the bottom is 10m away from the wall. If the ladder is pushed towards the wall at a rate of 0.25m per sec. Determine how fast is the top of the ladder moving up the wall at $t = 12\ sec.$

(7marks)

**c)** find $\frac{dy}{dx}$ if $xy + ln(x + y) = 1$

(5marks)

**d)** find $\frac{dy}{dx} if y = \cos(\cos x)$

(3marks)

**QUESTION THREE (20 marks)**

a) Find from the first principles, the derivative of the functions

(6marks)

i. $f(x) = sinx$

ii. $f(x) = \sqrt{x + 2}$

b) Find the derivatives of the following functions using appropriate methods

i) $y = \frac{x^2 + 3x - 4}{2x + 1}$ (4marks)

ii) $y = \cos(5x^3)$ (3marks)

c) Differentiate the implicit function;

d) $x^2 + 2y^3$ (4 marks)

**e)** find $\frac{dy}{dx} if y = \cos(\cos x)$ (3marks)

**QUESTION FOUR (20 marks)**

a) Given the function $g(y) = \begin{cases} y^2 + 5 \ \ if y < -2 \\ 1 - 3y if y \geq -2 \end{cases}$. Compute the $\lim_{y \to -2} g(y)$

(5marks)

b) At what point does the tangent to the function $y = x^3 + 2x^2$ have a slope of zero. (6marks)

c) Given that $P = 3q^4 - 4q^2 + 3;$ $\frac{d^3p}{dq^3}$

(4marks)

d) A church window with rectangular bottom and a semi-circular top is build using a 12m framing material. Determine the dimensions of the window to let in most light.

(5marks)

**QUESTION FIVE (20 marks)**

e) The amount of air in a balloon at any time t is given by $V(t) = \frac{6\sqrt[3]{t}}{4t+1}$. Determine if the balloon is being drained or being filled with air at $t = 8$.

(4marks)

f) Evaluate the derivative of the function $f(t) = \frac{t}{1+t}$ from the first principles

(4marks)

g) Differentiate $y = \tan^{-1} 3x^2$

(4marks)

a) **Eva**luate :$\lim \frac{\tan 2x}{x}$

**(**4marks)

**i.** The parametric equation of the curve are $x = e^t and \sin t . Find \frac{dy}{dx} \frac{d^2y}{dx^2}$

(4marks)