

**KISII UNIVERSITY**

**UNIVERSITY EXAMINATIONS 2022/2023**

**FIRST YEAR SEMESTER ONE EXAMINATION FOR THE AWARD OF THE  
DEGREE OF BACHELOR OF SCIENCE MATHEMATICS AND COMPUTING**

**MATH 102: DISCRETE MATHEMATICS**

**STREAM: Y1S1**

**TIME: 2 HOURS**

**INSTRUCTIONS**

1. Do not write anything on this question paper.
2. Answer question **ONE** and any other **TWO** questions.

**QUESTION ONE [COMPULSORY] (30 MARKS)**

- a) State the domain and range of the function represented by the following ordered points  
 $\{(2,6)(3,9)(4,12)(5,15)(6,18)\}$  (2 marks)
- b) Show that the following propositions are equivalent using a truth table  
 $P \Rightarrow (Q \Rightarrow R) \equiv (P \wedge Q) \Rightarrow R$  (8 marks)
- c) Define a function. (2 marks)
- d) Prove that  $\sqrt{2}$  is not a rational number. (8 marks)
- e) Find the negation of  $P \Rightarrow Q$  and hence or otherwise find the negation of the statement  
*He is rich and unhappy* (7 marks)
- f) Draw a Venn diagram to illustrate  $A \cup (B \cap C)$  (3 marks)

**QUESTION TWO (20 MARKS)**

- a) Using the universal set  $U = \{a, b, c, d, e, 1, 2, 3, 4, 5\}$ , set  $A = \{1, 2, 3\}$  and set  $B = \{a, b, c, 2, 5\}$ .
  - i) Prove that  $(A \cap B)^c = A^c \cup B^c$  (5 marks)
  - ii) Prove that  $(A \cup B)^c = A^c \cap B^c$  (5 marks)
- b) Write the converse, inverse and contrapositive statements of the following statement:  
*'If triangles are similar, then they are concurrent.'* (6 marks)
- c) Distinguish between a tautology and a contradiction. (4 marks)

**QUESTIN THREE (20 MARKS)**

- a) Test the validity of the following argument: (6 marks)  
*If a person is poor, he is unhappy*  
*If a person is unhappy, he dies young*  

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 *$\therefore$  a poor person dies young.*
- b) Prove that the following argument is a fallacy: (6 marks)  
*If I drive to work, I arrive early.*  

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*I did not drive to work.*

$\therefore$  I did not arrive early.

c) Prove by mathematical induction that

$$1 + 2 + \dots + n = \frac{n(n+1)}{2} \quad (8 \text{ marks})$$

**QUESTION FOUR (20 MARKS)**

a) Describe the conditions necessary for a function  $f$  to be regarded as:

i) Onto/ surjection (2 marks)

ii) One-to-one / injection (2 marks)

iii) Bijection / one-to-one correspondence (2 marks)

b) What are the three necessary conditions for a relation to be called an equivalence relation? (3 marks)

c) Find the domain and the range of the equation  $y = \frac{1}{(x+1)(x-3)}$ ;  $-1 \leq x \leq 3$

Is the equation above a relation or a function? (5 marks)

d) Show that  $n^3 + 2n$  is divisible by 3 for every positive integer  $n$ . (6 marks)

**QUESTION FIVE (20 MARKS)**

a) A test has 5 questions in Section A and 3 questions in Sections B. A student has to choose any 3 questions and 2 questions from sections A and B respectively and answer the questions in any order. Find how many ways the student

iv) Can choose the questions to answer (4 marks)

v) Can order the questions she answers (3 marks)

b) Find the negation of  $P \Rightarrow Q$  and hence or otherwise find the negation of the statement *He is rich and unhappy* (7 marks)

c) Given  $f(x) = 2x + 1$  and  $g(x) = 2x^2 + 1$

i) Find  $f \circ g(x)$  (3 marks)

ii) Evaluate  $f \circ g(-3)$  (3 marks)