KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATIONS FOR THE AWARD OF THE BACHELOR OF ART AND SCIENCE

SEMESTER ONE, 2022

MATH 111: CALCULUS 1

QUESTION 1 (30 mks)

a)

i)Investigate continuity of

$$f(x) = \begin{cases} x^2 + 2x, & x \leq -2 \\ x^3 - 6x, & x > 2 \, at \, the \, point \, x = -2 \end{cases}$$

(3 mks)

ii) Investigate continuity of

$$f(x) = \begin{cases} \frac{x-6}{x-3}, & x < 0 \\ 2, & x = 0 \\ \sqrt{4+x^2}, & x > 0 \, at \, x \, 0 \end{cases}$$ (3 mks)

b) i) Compute the following

$\lim_{x\to\infty} \frac{x^3+x-20}{x^2+4x+3}$ (3 mks)

ii) Compute the following

$\lim_{x\to 0} \frac{x^3-2x+\sqrt{\frac{3x^2}{4x}}+24x^3}{\frac{x}{4x-x^2}+\sin x}$ (3 mks)

c)

i) Solve the following trigonometric functions

$\sin(30° + 50°)$ (3 mks)

ii) Differentiate $y = \frac{X^4-X^3+\sqrt{x}-1}{X^2}$ (3 mks)

d)Total Cost (T.C) and Total Revenue (T.R) for a product are:

$T.C = \frac{1}{2}x^3 + \frac{5}{2}x^2 + 5x + 500$

$T.R = 800x - 2x^3$ where x is the number of units produced or sold.

Find:

i) Profit (P) in terms of x (3 mks)

ii) Profit maximizing quantity units (3 mks)

iii) Profit at point of maximizing quantity (3 mks)

iv) Average cost of profit maximizing quantity
(3 mks)

QUESTION 2

a) An average revenue (AR) and average cost (AC) functions for a given firm and product are:
$AR = 4 - \frac{1}{4}Q$
$AC = \frac{4}{Q} + 2 - 0.3Q + 0.05Q^2$
where Q is the number of units produced as sold.
Find an expression for:
i) Marginal revenue (M.R) (3 mks)
ii) Marginal cost (M.C) (3 mks)
iii) Profit (3 mks)
Find
i) Number of units that will maximize profit and minimum profit
(3 mks)
ii) Profit at unit maxima and minimum profit (3 mks)
b) Compute the following
i) $\lim_{x \to -2} x^3 - 2x + 1$ (3 mks)
ii) $\lim_{x \to 0} \frac{x^2}{4x - x^2}$ (3 mks)

QUESTION 3

a) i) Find equation of tangent and normal to a curve defined by $x = t^2 + 5t$ and $y = t^3$ at a point (-4, -1) (4 mks)
ii) Find equation at normal and tangents for curves at points given $3x^2 - 2xy + y^2 = 9$ at (-2, -3) (4 mks)
b) i) Find $\frac{d^2y}{dx^2}$ given
$x = (t^2 + 3)^{\frac{1}{2}}$ and $y = (3t + 4)^2$ (4 mks)
ii) Given $y = x^6 e^{2x}$ find $\frac{d^3y}{dx^3}$ (4 mks)
iii) Differentiate $y = x^3 \cos 3x \mid n\, x$ (4 mks)

QUESTION 4

a) Use the quotient rule to differentiate
i) $y = \frac{2e^t}{\sqrt{e^t + 4}}$ (4 mks)
ii) $y = \frac{3x}{(4x^2 - 1)^5}$ (4 mks)
b) Use the product rule to differentiate
i) $y = 2t.\sqrt{3t^2 + 1}$ (4 mks)
ii) $y = 3 \cos 5\Theta \sin 3\Theta$ (4 mks)
iii) $y = \cos^3 x sin^2 x$ (4 mks)