KISII UNIVERSITY
SCHOOL PURE AND APPLIED SCIENCES
DEPARTMENT MATHEMATICS AND ACTUARIAL SCIENCE
COURSE TITLE:FUNDAMENTAL OF ACTUARIAL MATHEMATICS II
COURSE CODE: BACS 210
FINAL EXAM DECEMBER 2022
INSTRUCTIONS:Answer question one and any other 2 questions in section B

SECTION A (30 marks)

Question One (30 marks)

(a) An investor pays 80 at the start of each month into a 25-year savings plan. The contributions accumulate at an effective rate of interest of 3% per half-year for the first 10 years, and at a force of interest of 6% per annum for the final 15 years. Calculate the accumulated amount in the savings plan at the end of 25 years. (5mks)

(b) At time $t = 0$ an investor purchased an annuity-certain which paid her £10,000 per annum annually in arrear for three years. The purchase price paid by the investor was £25,000. The value of the retail price index at various times was as shown in the table below:

Accident year	*Development year*		
	0	*1*	*2*
2017	2,440	3,294	3,788
2018	2,065	2,849	
2019	2,158		

Past and projected future inflation is given by the following index (measured at the midpoint of the relevant year).

Year	*Index*
2017	100
2018	105
2019	109
2020	116
2021	123

Calculate, to the nearest 0.1%, the following effective rates of return per annum achieved by the investor from her investment in the annuity: the real rate of return; and the money rate of return (10mks)

(c) The cumulative incurred claims for an insurance company for the last four accident years are given in the following table:

	Development year			
Accident year	*0*	*1*	*2*	*3*
2005	96	136	140	168
2006	100	156	160	
2007	120	130		
2008	136			

It can be assumed that claims are fully run off after three years. The premiums received for each year from 2005 to 2008 are 175, 181, 190 and 196 respectively. Calculate the reserve at the end of year 2008 using The Bornhuetter-Ferguson method. (5mks)

(d) The reserving department of a general insurance company has obtained the following incremental claims data (in 000*s*). You may assume that all claims are paid at the end of the year.

	Development Year			
Accident Year	*0*	*1*	*2*	*3*
2000	210	95	40	10
2001	225	105	45	
2002	215	95		
2003	220			

Underlying claims inflation rates over the twelve months to the middle of each year were as follows:

2001	3.0%
2002	2.5%
2003	2.5%

Claims inflation from the middle of 2003 onward is assumed to be 3.0% per annum.

(i) Calculate the outstanding claims reserve at 31 December 2003 using the inflation adjusted chain ladder. (5mks)

(e) An investor bought a number of shares at 78 each on 31 December 2005. She received dividends on her holding on 31 December 2006, 2007 and 2008. The rate of dividend per share is given in the table below:

	Development Year		
Accident Year	*1*	*2*	*3*
2016	130	180	190
2017	140	185	
2018	150		

On 31 December 2008, she sold her shares at a price of 93 pence per share.Calculate, using the retail price index values shown in the table, the effective annual real rate of return achieved by the investor. (5mks)

SECTION B

Question Two (20 marks)

(a) A university student receives a 3-year sponsorship grant. The payments under the grant are as follows:

. Year 1 £5,000 per annum paid continuously.

. Year 2 £5,000 per annum paid monthly in advance.

. Year 3 £5,000 per annum paid half yearly in advance.

Calculate the total present value of these payments at the beginning of the first year using a rate of interest of 8% per annum convertible quarterly. (10mks)

(b) Calculate the single premium payable for a temporary reversionary annuity of 12,000 per annum payable monthly in arrear to a female life currently aged 55 exact on the death of a male life currently aged 50 exact. No payment is made after 20 years from the date of purchase. (5mks)

(c) The reserving department of a general insurance company has obtained the following incremental claims data (in 000*s*). You may assume that all claims are paid at the end of the year.

	Development Year			
Accident Year	*0*	*1*	*2*	*3*
2000	210	95	40	10
2001	225	105	45	
2002	215	95		
2003	220			

Underlying claims inflation rates over the twelve months to the middle of each year were as follows:

2001	3.0%
2002	2.5%
2003	2.5%

Claims inflation from the middle of 2003 onward is assumed to be 3.0% per annum.

(i) Calculate the outstanding claims reserve at 31 December 2003 using the inflation adjusted chain ladder. (5mks)

Question Three (20 marks)

(a) The force of interest $\delta(t)$ is a function of time and at any time t, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.04 & \text{if } 0 \leq t < 5 \\ 0.008t & \text{if } 5 \leq t < 10 \\ 0.005t + 0.0003t^2 & \text{if } t \geq 10 \end{cases}$$

Calculate the present value at time $t = 0$ of a continuous payment stream that is paid at the rate of $e^{-0.05t}$ per unit time between time $t = 2$ and time $t = 5$. (5mks)

(b) In its premium rate basis, an office assumes a 3-year select period. The table is such that: $q_{[x+7]} = q_{([x+3]+1)} = q_{([x]+2)} = q_{(x+1)}$ and ultimate mortality follows A1967-70 ultimate. Calculate $l_{[x]+t}$ for $t = 0, 1$ and 2. (5mks)

(c) The cumulative incurred claims for an insurance company for the last four accident years are given in the following table:

	Development year			
Accident year	*0*	*1*	*2*	*3*
2005	96	136	140	168
2006	100	156	160	
2007	120	130		
2008	136			

It can be assumed that claims are fully run off after three years. The premiums received for each year from 2005 to 2008 are 175, 181, 190 and 196 respectively. Calculate the reserve at the end of year 2008 using The Bornhuetter-Ferguson method. (5mks)

(d) A life aged exactly 60 wishes to arrange for a payment to be made to a charity in 10 years' time. If he is still alive at that date the payment will be 1000. If he dies before the payment date, the amount given will be 500. Assuming an effective interest rate of 6% per annum and mortality according to ELT No.12-Males, calculate the standard deviation of the present value of the liability. (5mks)

Question Four (20 marks)

(a) The force of interest $\delta(t)$ is a function of time and at any time, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.04 + 0.01t & \text{if } 0 \leq t < 4 \\ 0.12 - 0.01t & \text{if } 4 \leq t < 8 \\ 0.06 & \text{if } t \geq 8 \end{cases}$$

Calculate the present value at time $t = 0$ of a payment stream, paid continuously from time $t = 9$ to $t = 12$, under which the rate of payment at time t is $50e^{0.01t}$. (5mks)

(b) The table below shows the incremental claims paid together with an extract of index of prices. Claims are fully paid by the end of development year 3.

Accident year\development year	0	1	2	3	Year	Price Index (mid year)
2006	103	32	29	13	2006	100
2007	88	21	16		2007	104
2008	110	35			2008	109
2009	132				2009	111

(i) Calculate the reserve for unpaid claims using the inflation-adjusted chain ladder, assuming that future claims inflation will be 3% p.a. (5mks)

(c) Find expressions in terms of the life table functions $l_{[x]+t}$ and l_y for $q_{[50]}$, $_{2|}q_{[50]}$, $_{2|3}q_{[50]+1}$ (5mks)

(d) Calculate the annual premium payable by a man aged 32 for a temporary assurance with a sum assured of $5,000$ and a term of 12 years. Assume AM92 Ultimate mortality and 4% pa interest. (5mks)

Question Five (20 marks)

(a) The cost of claims incurred each year is given in the table below:

	Development Year		
Accident Year	*0*	*1*	*2*
1999	2,317	1,437	582
2000	3,287	1,792	
2001	4,816		

The cumulative number of reported claims is shown in the table below:

	Development Year		
Accident Year	*0*	*1*	*2*
1999	132	197	207
2000	183	258	
2001	261		

Given that the total claims paid to date is 10,237 calculate the outstanding claims reserve for this cohort using the average cost per claim method. (5mks)

(b) The mortality of a certain population is governed by the life table function $l_x = 100 - x, \quad 0 \leq x \leq 100$. Calculate the values of the following expressions: $_{10}p_{30}$ and μ_{30} (5mks)

(c) An annuity provides payments of 40 at the end of each month forever. If the interest rate is 10 percent pa convertible quarterly, calculate the present value of the annuity. (5mks)

(d) A life aged 65 is assumed to be subject to an annual initial rate of mortality equal to twice that of the $AM92$ Ultimate tables for the next two years. Calculate the probability that the life will die before age 67. (5mks)