KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE AWARD OF

THE DEGREE OF BACHELOR OF SCIENCE, EDUCATION, COMPUTER SCIENCE AND MATHEMATICS

FIRST SEMESTER 2022/2023

SEPTEMBER-DECEMBER, 2022

MATH 210: LINEAR ALGEBRA 1

STREAM: Y2 S1 **TIME: 2 HOURS**

DAY: **DATE:**

INSTRUCTIONS

1. ***Do not write anything on this Question Paper***
2. ***Answer Question ONE and Any Other TWO Questions.***
3. ***All workings should be shown in the examination booklet. Rough work should be crossed out.***

QUESTION ONE (30 MARKS)

a) Find the eigen values and the associated eigen vectors for the matrix [4 Marks]

$$A = \begin{bmatrix} 0.5 & 2 \\ 1 & 1.5 \end{bmatrix}$$

b) i) State cramer's rule [2 Marks]

ii) Solve the system $Ax = b$ using cramer's rule [4 Marks]

$$x_1 + 4x_2 + 3x_3 = 10$$

$$2x_1 + x_2 - x_3 = -1$$

$$3x_1 - x_2 + x_3 = 11$$

c) i) Define an inconsistent system. [2 Marks]

ii) Find the solution of the set $x_1 - 4x_2 + 7x_3 = 6$ [3 Marks]

d) i) Define the adjoint of a given matrix A [2 Marks]

ii) Find the adjoint of [4 Marks]

$$A = \begin{bmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

e) Reduce the following matrix to upper triangular form [4 Marks]

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 1 & 2 \end{bmatrix}$$

f) Calculate the area of a triangle whose vertices are $A(3,0)$, $B(6,6)$, $C(12,9)$ and briefly comment on your answer. [4 Marks]

g) Find the trace of the matrix. [3 Marks]

$$\begin{bmatrix} -3 & 21 & 0 & 3 \\ 0 & 15 & 6 & 3 \\ 3 & 0 & 3 & 6 \\ 0 & 12 & 12 & 24 \end{bmatrix}$$

QUESTION TWO (20 MARKS)

a) Use Gauss elimination method to solve. [5 Marks]

$$x - 2y + 3z = 1$$

$$x + y + 4z = -1$$

$$2x + 5y + 4z = -3$$

b) i) Consider the vectors $\underset{\sim}{u} = (2, -6, 14)$ and $\underset{\sim}{v} = (16, -4, -4)$. Find $\underset{\sim}{u} \cdot \underset{\sim}{v}$ and the angle between them. [4 Marks]

ii) Comment on your answer in b(i). [1 Marks]

c) Let $\underset{\sim}{u} = (1,2,-1)$ and $\underset{\sim}{v} = (6,4,2)$ be vectors in $\mathbb{R}^3$, Show that $\underset{\sim}{w} = (9,2,7)$ is a linear combination of $\underset{\sim}{u}$ and $\underset{\sim}{v}$. [5 Marks]

d) State the Cauchy-Schwartz inequality for the inner product and verify it for the vectors $\underset{\sim}{u} = (3,1)$ and $\underset{\sim}{v} = (-1,2)$ using Euclidean inner product. [5 Marks]

QUESTION THREE (20 MARKS)

a) Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a linear operator defined by $T(x, y, z) = (x + 2y - z,\ \ y + z,\ x + y - 2z)$. Find the basis and dimension of

i) $Ker(T)$

ii) $Range\ of\ T$ [6 Marks]

b) Solve by Gauss-Jordan elimination. [6 Marks]

$$\frac{x_1}{2} - 2x_2 - x_3 = \frac{21}{2}$$

$$x_1 + \frac{x_2}{2} + x_3 = \frac{3}{2}$$

$$\frac{3}{2}x_1 + x_2 - \frac{x_3}{2} = 1$$

c) Find the characteristic polynomial of matrix A given by [4 Marks]

$$A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$$

d) Transform $\begin{bmatrix} 1 & 3 & 3 \\ 2 & 4 & 10 \\ 3 & 8 & 4 \end{bmatrix}$ into a unit matrix. [4 Marks]

QUESTION FOUR (20 MARKS)

a) State the Cayley-Hamilton theorem and use it to find the characteristic equation the matrix [6 Marks]

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -2 & 2 \end{bmatrix}$$

b) Determine whether or not the following sets of point are collinear [4 Marks]

i) (1,2) (3,4) (5,6)

ii) (1,0) (1,1) (3,3)

c) Let $V = \mathbb{R}^3$ with standard operations and $S = \{(1,2,3), (0,1,2), (-2,0,1)\} \leq \mathbb{R}^3$. Does S span V. [4 Marks]

d) i) Define the term diagonalizable matrix. [2 Marks]

ii) Show that $B = \begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}$ is not diagonalizable. [4 Marks]

QUESTION FIVE (20 MARKS)

a) Given [6 Marks]

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -1 & 1 \\ 4 & 2 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

By use of co-factor method solve for x, y, z, given $AX = C$

b) Find the angle between two vectors $\underset{\sim}{u} = (3, 2, -1)$ and $\underset{\sim}{v} = (2,2,3)$ by using the

i) Euclidean inner product [2 Marks]

ii) Inner product given by $\langle u, v \rangle = 2u_1v_1 + 3u_2v_2 + u_3v_3$ [4 Marks]

c) Find the rank of $M = \begin{bmatrix} 1 & 0 & 3 & 0 & 4 & 0 \\ 2 & 1 & 7 & 0 & 10 & 1 \\ 1 & 0 & 3 & 2 & 6 & 4 \\ 1 & 0 & 3 & 0 & 4 & 0 \end{bmatrix}$ [4 Marks]

d) Given $\underset{\sim}{u} = (1,\ 0, -2)$ and $\underset{\sim}{v} = (-3, 5, 1)$. Find the orthogonal projection of $\underset{\sim}{u}$ on $\underset{\sim}{v}$ and the component of vector $\underset{\sim}{u}$ orthogonal to $\underset{\sim}{v}$. [4 Marks]

e) Show that the determinant of a second-order matrix with identical row is zero. [2 Marks]