



UNIVERSITY EXAMINATIONS

**SECOND YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF
BACHELOR OF SCIENCE IN MATHEMATICS AND ACTUARIAL SCIENCE**

**FIRST SEMESTER 2022/2023
[SEPTEMBER-DECEMBER, 2022]**

MATH 211: CALCULUS II

STREAM: Y2S1

TIME: 2 HOURS

DAY: MONDAY, 3:00 -5:00 PM

DATE: 19/12/2022

INSTRUCTIONS

1. Do not write anything on this question paper.
2. Answer question ONE and any other TWO questions.

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SECTION A (30 MARKS)

1.

a. Integrate $\int_1^2 x \ln x dx$ (5 marks)

b. Use integration by substitution to solve $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\sqrt{1-y^2}} dy$ (5 marks)

c. Use integration by partial fractions to solve $\int \frac{2x^3-4x^2-x-3}{x^2-2x-3} dx$ (5 marks)

d. Use integration by parts to solve $\int_0^\pi [x^3 \cos x] dx$ (5 marks)

e. Find the Taylor series for

$f(x) = \sin 2x$ at $x_0 = 0$ to the 7th approximation (5 marks)

f. Integrate $\int_0^1 \frac{dt}{t^2-6t+10}$ (5 marks)

SECTION B (40 MARKS)

2.

- a. Find $\int_0^e \frac{\sqrt{1+\ln x}}{x} dx$ (5 marks)
(5 marks)
- b. Find the area between the x axis, the curve $y = \frac{1}{x}$ and the lines $x = -e^3$ and $x = -e$. (5 marks)
- c. State the Mean Value Theorem and check if it is satisfied in $\frac{x^2-5x}{x-3}$, on $[0,5]$ (5 marks)

3.

- a. Let $P(t)$ denote the population of bacteria in a certain colony at time t . Suppose that $P(0) = 100$ and that P is increasing at a rate of $20e^{3t}$ bacteria per day at time t . How many bacteria are there after 50 days? (5 marks)
- b. Evaluate $\int_0^1 \sin^3 x \cos x dx$ (5 marks)
- c. Integrate $\int \left\{ \frac{1}{x} + \sin\left(\frac{1}{4}x\right) + \sqrt{4x} - e^{-3x} + 4^x - \frac{6x}{3x^2-5} + \ln 2x \right\} dx$ (10 marks)

4.

- a. Find if $\lim_{x \rightarrow 0} \frac{\tan 3x}{\tan 2x}$ exists (5 marks)
- b. Solve $\int_0^{\frac{\pi}{2}} \cos 3x \sin 2x dx$ (5 marks)
- c. State Rolle's Theorem (5 marks)
- d. State the Rolle's Theorem. Hence verify that the function $f(x) = x^2 - 4x + 3$ satisfies the conditions of Rolle's Theorem. (5 marks)

5.

- a. Differentiate giving examples between definite and indefinite integration. (5 marks)
- b. Find the area between the graphs of $\cos x$ and $\sin x$ on $[0, \frac{\pi}{4}]$ (5 marks)
- c. Evaluate $\int_{-\pi}^0 \sin^4 x dx$ (5 marks)
- d. $\int \frac{x}{1+x^4} dx$ (5 marks)