KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL/APPLIED STATISTICS/EDUCATION/COMPUTER SCIENCE

FIRST SEMESTER 2022/2023

[SEPTEMBER-DECEMBER, 2022]

MATH 220: VECTOR ANALYSIS

STREAM: Y2S1 **TIME: 2 HOURS**

DAY: WEDNESDAY, 9:00 – 11:00 AM **DATE: 21/12/2022**

INSTRUCTIONS

1. ***Do not write anything on this question paper.***
2. ***Answer question ONE and any other TWO questions.***

QUESTION ONE (COMPULSORY) **30MKS**

a) Given that $\vec{A} = (2x^2y - x^4)\mathbf{i} + (e^{xy} - y\sin x)\mathbf{j} + (x^2\cos y)\mathbf{k}$. Find $\frac{\partial^2 \vec{A}}{\partial x\, \partial y}$ at (1,1) (3mks)

b) Determine the curl of vector $\vec{F}$ at the point (2, 0, 3) given that $\vec{F} = \mathbf{i}ze^{xy} + \mathbf{j}2xz\cos y + (x + 2y)\mathbf{k}$. (4mks)

c) Show that the curl of $(-y\mathbf{i} + x\mathbf{j})$ is a constant vector (3mks)

d) Given that $\varphi = x^2\sin z + z\,e^y$, find the value of $|\boldsymbol{grad}\ \varphi|$ at point (1,3,2) (3mks)

e) If $\vec{A} = x^2z\mathbf{i} + xy\mathbf{j} + y^2z\mathbf{k}$ and $\vec{B} = yz^2\mathbf{i} + xz\mathbf{j} + x^2z\mathbf{k}$. Determine the expression for grad $(\vec{A}.\vec{B})$ at (1,1,1) (4mks)

f) A particle moves in space so that at time t its position is stated as $x = 2t + 3, y = 2t^2 + 3t, z = t^3 + 2t^2$. Find the component of velocity in the direction $2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ (4mks)

g) If If $\vec{F} = x^2y^2\mathbf{i} + y^3z\mathbf{j} + z^2\mathbf{k}$. Evaluate $\int_c \vec{F}.dr$ along the curve $x = 2u^2,\ y = 3u$ and $z = u^3$ between $A(2, -3, -1)$ and $B(2, 3, 1)$ (5mks)

h) Find the directional derivative of $\emptyset = x^2y - 2xz^2 + y^2z$ at the point (1,3,2) in the direction of the vector $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ (4mks)

QUESTION TWO (20MKS)

a) Given the vectors $\vec{A} = 2i + 3j + 5k \; and \; \vec{B} = -i + 4j + 6k, C = 2i + 4j - 3k$ determine

i) A unit vector perpendicular to $\vec{A} \; and \; \vec{B}$

ii) the magnitude of $(\vec{A} \times \vec{B}) \times \vec{C}$ (10mks)

b) Afield $\varphi = 1 + 2xy + x^2 - y^2 + z$ exists in a region of space. Determine, at point (1, -1, -2):

i) Grad φ

ii) Divgradφ

iii)the directional derivative of φ in the direction of the vector $\vec{A} = i + 2j - 2k$ (10mks)

QUESTION THREE (20MKS)

a) If $\vec{F} = (x^2y)\mathbf{i} + yz\mathbf{j} - 2yz\mathbf{k}$. Evaluate $\int_c \vec{F}.d\mathbf{r}$ from (0,0,0) to (4,2,1) along the path given by x = 4t , $y = 2t^2$ and $z = t^3$ (6mks)

b) A fluid motion is given by $\vec{v} = (y + z)\mathbf{i} + (z + x)\mathbf{j} + (x + y)\mathbf{k}$.Show that the motion is irrotational and hence find the velocity potential (8mks)

c) If $\vec{F} = 2\mathbf{i} + 4u\mathbf{j} + u^2\mathbf{k}$ and $\vec{G} = u^2\mathbf{i} - 2u\mathbf{j} + 4\mathbf{k}$. Find $\int_0^2 (\vec{F} \times \vec{G}) du$ (6mks)

QUESTION FOUR (20MKS)

a) Show that $\vec{r} = e^{-t}(c_1\cos 2t c_2 \sin 2t)$ where $c_1$and c_2 are constant vectors is a solution to the differential equation $\frac{d^2r}{dt^2} + 2\frac{dr}{dt} + 5r = 0$ (6mks)

b) If a force $\vec{F} = 2x^2y\mathbf{i} + 3xy\mathbf{j}$ displaces a aparticle in the x-y plane from (0,0) to (1,4) along a curve $y = x^2$. Find the work done (6mks)

c) Use divergence theorem to evaluate the $\iint \vec{F}.\hat{n} \; ds$ where$\vec{F} = (3x - 2z)\mathbf{i} - (2x + y)\mathbf{j} + (y^2 + 2z)\mathbf{k}$ and S is the surface of the sphere with centre at (1,2,4) radius 4 units (8mks)

QUESTION FIVE (20MKS)

a) If $\vec{r} = x\mathbf{i} + \mathbf{y}\mathbf{j} + z\mathbf{k}$, show that $\nabla^2(r^n) = n(n-1)r^{n-2}$ (6mks)

b) i) state stokes theorem (2mks)

ii) Use stock's theorem to evaluate$\int[(2x - y)dx - (yz^2)dy - (y^2z)dz]$ where S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and C is boundary of the sphere (6mks)

c) Verify the Green's theorem in the plane for $\oint_c [(3x - 8y^2)dx + (4y - 6xy)dy]$ where C is the square formed by lines $x = \pm 1, y = \pm 1$ (6mks)