KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

THIRD YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTING/ECONOMICS AND STATISTICS

FIRST SEMESTER 2022/2023

[SEPTEMBER-DECEMBER, 2022]

BACS 310: ACTUARIAL MATHEMATICS II

STREAM: Y3S1 **TIME: 2 HOURS**

DAY: WEDNESDAY, 3:00 – 5:00 PM **DATE: 07/12/2022**

INSTRUCTIONS

1. ***Do not write anything on this question paper.***
2. ***Answer question ONE and any other TWO questions.***

QUESTION ONE (30 MARKS)

1. Discuss two approaches to the theory of multiple-decrement tables.

(4marks)

2. Given the below multiple decrement table

Age x	no. Of lives $(al)_x$	heart disease $d_x^{(1)}$	accident $d_x^{(2)}$	other causes $d_x^{(3)}$
50	4832555	5168	1157	4293
51		5363	1206	5162
52		5618	1443	5960
53		5929	1679	6840
54		6277	2152	7631

a) Compute $(al)_x, (ad)_x, (aq)_x, (ap)_x, (aq)_x^1, (aq)_x^2, (aq)_x^3$ **(4marks)**

b) Calculate the present value of the benefits for a 3 year term policy to (50) whose benefits are paid at the end of year of exit. Policy pays 100/- at the end of year of death from any cause, an extra 100/- at the end of year of death from accidents, i=5%

(6marks)

3. Let β_1, β_2 be the modes of decrement in a double-decrement table. Suppose that β_1 is uniformly distributed over the year of age from x to x + 1 in its associated single- decrement table, and $\mu_{x+t}^{\beta_2} = c \; for \; 0 < t <$

1 Find formulae for $aq_x^{\beta_1}$ and $aq_x^{\beta_2}$ in-terms of $q_x^{\beta_1}$ and $q_x^{\beta_2}$

(6marks)

4. Write the Kolmogorov's Forward Equations (for the general case in which there are n states) given a system for each fixed x:

(3marks)

5. A friendly society issued a policy providing the following benefits to a man aged exactly 25 at entry:
   a) On death at any time before age 60, the sum of $4,000 payable immediately;
   b) On survival to age 60, an annuity of $8 per week payable weekly in advance for as long as he survives;
   c) On sickness, an income benefit to be payable during sickness of $32 per week for the first 6 months reducing to $16 per week for the next 18 months and to $8 per week thereafter. Sickness benefit is not payable after age 60. There is no waiting period. Premiums are payable monthly in advance for at most 35 years, and are not waived during periods of sickness.

   The society uses the following basis to calculate premiums. Find the monthly premium.
   mortality: English Life Table No.12 - Males
   sickness: Manchester Unity Sickness Experience 1893-97, Occupation Group AHJ
   interest: 4% p.a.
   expenses: none **(7marks)**

QUESTION TWO (20MARKS)

1. You are using the three-state illness-death model to price various sickness policies. Write down an expression for the expected present value of each of the following sickness benefits for a healthy life aged 30.
   a) £3,000 *pa* payable continuously while ill, but ceasing at age 60.
   b) £3,000 *pa* payable continuously throughout the first period of illness only, but ceasing at age 60.
   c) £3,000 *pa* payable continuously while ill provided that the life has been ill for at least one year. Again, any benefit ceases to be paid at age 60.

(10marks)

2. Suppose that in a triple-decrement model you are given constant forces of decrement for a person now age x, as follows:

   $\mu^{(1)}_{x+t} = b$, for $t \geq 0$

   $\mu^{(2)}_{x+t} = b$, for $t \geq 0$

   $\mu^{(3)}_{x+t} = 2b$, for $t \geq 0$

further, the probability that x will exit the group within 3 years due to decrement 1 is 0.00884 compute the length of time a person now age x is expected to remain in the triple decrement table

(10marks)

QUESTION THREE (20MARKS)

1. In a triple decrement table you are given that decrement (1) is death, decrement (2) is withdrawal. In addition:
   a) $q_{60}^{(1)}=0.01$ $q_{60}^{(2)}=0.05$ $q_{60}^{(3)}=0.10$
   b) Withdrawal occur only at the middle of the year
   c) Mortality and disability are uniformly distributed over each year of age in the associated single decrement tables.

   Calculate $(aq)_{60}^{(3)}$ **(10marks)**

2. The members of a large company's manual workforce are subject to three modes of decrement, death, withdrawal and promotion to supervisor. It is known that these workers' independent rates of mortality are those of English Life Table No. 12 - Males, the independent withdrawal rate is 0.03 at each age, and their independent promotion rate is 0.01 at age 50 and 0.02 at age 51.
   a) Draw up a service table for manual workers from age 50 to age 51 with a radix of 100,000 at age 50, including the value of $(al)52$.
   (b) Calculate the probability that a life aged exactly 50 will gain promotion within 2 years.

(10marks)

QUESTION FOUR (20MARKS)

1. Give a mathematical definition of the Markov property. **(3marks)**
2. Define a multiple decrement life-table giving 5 examples where it is applied insurance. **(5marks)**
3. A life office issues policies to lives aged under 60 providing the following benefits:
   (i) on becoming permanently disabled before age 60, an annuity of $2,000 per annum payable weekly for life and $20,000 immediately on death, and
   (ii) immediately on death before age 60 while not permanently disabled, $20,000.
   Calculate the office annual premium, payable weekly and ceasing on death, on permanent disability or on reaching age 60, for a life aged 58 if the office uses the following basis:
   Mortality: the independent rates of mortality of those not permanently disabled are those of A1967-70 ultimate; the permanently disabled are subject to the mortality of English Life Table No.12 - Males with the age rated up by 6 ½ years; Permanent disability: a constant independent rate of

0.006; Interest: 4% per annum; Expenses: 2 1/2% of all office premiums, plus $50 at the issue date. **(12marks)**

QUESTION FIVE (20 MARKS)

1. Under UD of D assumption between ages x and x+1 in each mode of decrement in a single-decrement table. Proof

(10marks)

(i) $aq_x^{\beta_1} = q_x^{\beta_1}(1 - 1/2q_x^{\beta_2})$

(ii) $(a\mu)_x = \mu_x^{\beta_1} + \mu_x^{\beta_2}$

(iii) $am_x^{\beta_1} \cong \mu_{x+1/2}^{\beta_1}$

(iv) $aq_x^{\beta_1} = q_x^{\beta_1}[1 - \frac{1}{2}(q_x^{\beta_2} + q_x^{\beta_3}) + \frac{1}{3}q_x^{\beta_2}.q_x^{\beta_3}]$

2. A certain variety of tomato is susceptible to blight, which is always fatal. A researcher decides to model the life cycle of the tomato using a multiple state model with the following states:

1. Not suffering from blight
2. Suffering from blight
3. Dead.

The transition rates are dependent on the age of the plant and are as follows:

μ_x is the mortality rate at exact age x of a blight-free plant

σ_x is the rate of contracting blight at exact age x

τ_x is the mortality rate at exact age x of a plant suffering from blight.

(i) Draw and label a transition diagram for this multiple state model.

Let $p_{ij}(x, y)$denote the probability that a plant is in State j at age y (y > x) given that it was in State i at age x .

(ii) Write down an expression involving transition rates for each of the following probabilities:

$$p_{11}(x, x + t)$$
$$p_{22}(x, x + t)$$

(iii) Write down an integral expression for $p_{12}(x, x + t)$ in terms of transition rates and the probabilities in (ii). **(10marks)**