



KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

**FOURTH YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF
BACHELOR OF SCIENCE IN MATHEMATICS AND EDUCATION**

FIRST SEMESTER 2022/2023

[SEPTEMBER-DECEMBER, 2022]

MATH 420: PARTIAL DIFFERENTIAL EQUATIONS I

STREAM: Y4S1

TIME: 2 HOURS

DAY: MONDAY, 12:00 – 2:00 PM

DATE: 05/12/2022

INSTRUCTIONS

1. *Do not write anything on this question paper.*
2. *Answer question ONE and any other TWO questions.*

QUESTION ONE

a) Find the integral surface of the quasi – linear partial differential equation $x(y^2 + z) p - y(x^2 + z)q = (x^2 - y^2)z$ which contains the straight line $x + y = 0, z = 1$ (5 Marks)

b) Show that the sphere equation $x^2 + y^2 + z^2 = a^2$ has parametric equations $x = \frac{a(1-v^2)}{1+v^2}$
 $\cos u, y = \frac{a(1-v^2)}{1+v^2} \sin u$ and $z = \frac{2av}{1+v^2}$ (6 Marks)

c) A curve is formed by the intersection two surfaces;

$F(x, y, z) = 3x^2y + y^2z + 2 = 0$ and $G(x, y, z) = 2xz - x^2y - 3 = 0$. Find the equation of the

- i. Normal plane.
- ii. Tangent line to that circle at $p(2, -1, -1)$ (7 Marks)

d) Find the integral curves of the equations;

$$\frac{dx}{mz-ny} = \frac{dy}{nx-lz} = \frac{dz}{ly-mx} \quad (6 \text{ Marks})$$

e) Find the orthogonal trajectories on the cone $x^2 + y^2 = z^2 \tan^2 \alpha$ of its intersections with the family of planes parallel to $z = c$ (6 Marks)

QUESTION TWO

- a) Find the integral curves of the equations. (8marks)

$$\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2}$$

- b) Verify that the equation $x(y^2 - a^2)dx + y(x^2 - z^2)dy - z(y^2 - a^2)dz = 0$ is integrable and hence find its solution (7Marks)

- c) Solve the pfaffian differential equation.

$$(y + y^2 x^2) dx = x dy \quad (5Marks)$$

QUESTION THREE

- a) Find the surface which intersects with the surfaces of the system $z(x + y) = c(3z + 1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1, z = 1$ (10 Marks)

- b) Test the integrability of $(x^2z - y^3)dx + 3xy^2 dy + x^3 dz = 0$ (6 Marks)

- c) Find the angle between $A = 2i - 3j + 6k$ and $B = -i + 4j - 2k$ (4 Marks)

QUESTION FOUR

- a) For the parametric equation

$$x = u + v$$

$$y = u - v$$

$$z = 4uv$$

Prove that they represent a surface and hence find its associate implicit equation. (7 Marks)

- b) Use cauchy's method to solve the equation

$$z = \frac{1}{2}(P^2 + Q^2) + (P-x)(Q-y) \quad (13 Marks)$$

QUESTION FIVE

- a) Use Jacobi method to find the compatible solution of $p^2x + Q^2 + Q^2y = z$ (12 Marks)

- b) Find the integral surface of the partial differential equation $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ which passes through straight line $x + y = 0$ (8 Marks)