KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

FOURTH YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF BACHELOR OF EDUCATION SCIENCE
FIRST SEMESTER 2022/2023
[SEPTEMBER-DECEMBER, 2022]

PHYS 411: QUANTUM MECHANICS

STREAM: Y4S1 **TIME: 2 HOURS**

DAY: FRIDAY, 9:00 – 11:00 AM **DATE: 23/12/2022**

INSTRUCTIONS

1. Do not write anything on this question paper.
2. Answer Question ONE and any other TWO Questions.

QUESTION ONE [30 MARKS]

a) By considering anisotropic oscillator in a three dimensional potential, write the eigen energies corresponding to the potential. [2 Marks]

b) Define the term Hermitian operator and using the wave function ($\boldsymbol{\psi}$) gives its mathematical expression. [2 Marks]

c) Taking

$\hat{J}_{\pm} = \hat{J}_x \pm i\hat{J}_y$

Show that:

$[\hat{J}_+, \hat{J}_-] = 2\hbar\hat{J}_z$

[4 Marks]

d) Derive the following commutation relationship: [4 Marks]

i. $[L_+, L_Z] = -\hbar L_+$

ii. $[L_-, L_Z] = -\hbar L_-$

e) Given that the electrons total momentum vector is given by; [4 Marks]

$J = LxS$ and that $L \times L = i\hbar L$ and $S \times S = i\hbar s$

Find $J \times J$

f) What problem does stationary perturbation theory try to solve in quantum mechanics. [2 Marks]

g) Differentiate between degenerate perturbation theory and non-degenerate perturbation theory. [4 Marks]

h) Prove that;

$J^2 = L^2 + S^2 + 2L.s$ [4 Marks]

i) Write $L_x, L_y\ and\ L_z$ in the spherical polar coordinates using r, $\theta\ and\ \varphi$. [3 Marks]

QUESTION TWO [20 MARKS]

a) show that for a quantum mechanical oscillator the harmonic oscillator eigen function is given by: [10 Marks]

$$E_n = \left(n + \frac{1}{2}\right)\hbar\boldsymbol{\omega}$$

b) systematically show that the time independent schrodinger equation in spherical coordinates takes the form: [10 Marks]

$$\left[-\frac{\hbar^2}{2m}\frac{1}{r}\frac{\partial^2}{\partial r^2}r + \frac{1}{2mr^2}L^2 + V_{(r)}\right]\Psi(\mathrm{r}) = \mathrm{E}\Psi(\mathrm{r})$$

QUESTION THREE [20 MARKS]

a) Systematically show that in the degenerate perturbation theory, the first order correction in the energy is given by; [10 Marks]

$$\sum_{\alpha=1}^{f}(\hat{H}_{P,\beta\alpha}\text{-}E_n^{(1)}\delta_{\alpha,\beta})\mathrm{a}\alpha\text{=}0$$

Where $\beta = 1,2,3, \dots \dots ....\mathrm{f}$

b) In the first oder correction in energy E_n^1 show that; [10 Marks]

$$I\psi_n >= I\varphi_n >= \sum_{m\neq n}\frac{\langle\varphi_m|\hat{H}_p|\varphi_n\rangle}{E_n^{(0)} - E_m^{(0)}}$$

QUESTION FOUR [20 MARKS]

a) What are the goals of time dependent perturbation theory. [3 Marks]

b) Show that; [6 Marks]

$$\sum_n i\hbar\frac{\partial c_n}{\partial t} - c_n(t)V(t)e^{\frac{-iE_nt}{h}}In >= 0$$

c) What are the main differences between time independence perturbation theory and time dependent perturbation theory. [4 Marks]

d) For time dependent perturbation theory, show that; [7mks]

$$i\hbar\frac{dc_n(t)}{dt} = \sum_m H_{nm}(t)\exp(i\boldsymbol{\omega_{nm}t})\,\boldsymbol{C_m(t)}$$

QUESTON FIVE [20 MARKS]

a) A particle of charge q and mass M, which is moving in one dimension. Harmonic potential of frequency ω is subjected to a weak electric field E in the x-direction.

   i) Find the expression for the energy. [5 Marks]

   ii) Calculate the energy to the first non-zero correction and compare it to the exact result obtained in (a) [10 Marks]

b) By clearly showing each step, prove that the following equation are correct;

   $[L^2, L_x] = [L^2, L_y] = [L^2, L_z] = 0$ [5 Marks]