KISII UNIVERSITY
SCHOOL PURE AND APPLIED SCIENCES
DEPARTMENT MATHEMATICS AND ACTUARIAL SCIENCE
COURSE TITLE:ELEMENT OF RISK AND INSURANCE
COURSE CODE: BACS 120
FINAL EXAM APRIL 2021
INSTRUCTIONS:Answer question one and any other 2 questions in section B

SECTION A (30 marks)

Question One (30 marks)

(a) The Gamma distribution with mean μ and variance $\frac{\mu^2}{\alpha}$ has density function

$$f(y) = \frac{\alpha^{\alpha}}{\mu^{\alpha}\Gamma(\alpha)} y^{\alpha-1} e^{-\frac{y\alpha}{\mu}} \qquad (y > 0)$$

Show that this may be written in the form of an exponential family. (5mks)

(b) Prove that given $\frac{G'(S)}{G(s)} = \frac{(a+b)}{1-as}$ it follows Geometric distribution distribution. (5mks)

(c) Consider a zero-coupon corporate bond that promises to pay a return of 10% next period. Suppose that there is a 10% chance that the issuing company will default on the bond payment, in which case there is an equal chance of receiving a return of either 5% or 0%.

(i) Calculate values of downside semi-variance and shortfall probability based on the risk-free rate of return of 6% (5mks)

(d) The entries for age 60 in a multiple decrement table with decrements for death and retirement are as follows: If $q^d_{62} = 9,000$, $q^d_{61} = q^d_{60}$ and decrements operate

Age	$(al)_x$	$(ad)^d_x$	$(ad)^r_x$
60	10,000	200	400

uniformly over the year of age in the multiple decrement table, calculate the value of q^r_{61}. (5mks)

(e) The number of claims on a portfolio of washing machine insurance policies follows a Poisson distribution with parameter 50. Individual claim amounts for repairs are a random variable $100X$ where X has a distribution with probability density function

$$f(x) = \begin{cases} \frac{3}{32}(6x - x^2 - 5) & \text{if } 1 < x < 5 \\ 0 & \text{otherwise} \end{cases}$$

(i) Calculate the mean and variance of the total individual claim amounts. (7mks)

(ii) Calculate the mean and variance of the aggregate claims . (3mks)

SECTION B

Question Two (20 marks)

(a) A multiple decrement table is subject to 3 modes of decrement: α, β and γ. You are given the following extract from the table:

Age, x	$(al)_x$	$(ad)^\alpha_x$	$(ad)^\beta_x$	$(ad)^\gamma_x$
50	5,000	86	52	14
51	4,848	80	56	20

(i) Calculate the probability that a 50-year old leaves the population through decrement γ between the ages of 51 and 52. (3cm)

(ii) Assuming that each decrement is uniformly distributed between integer ages in the multiple decrement table, calculate the independent probabilities q_{50}^{α} and $_{1|}q_{50}^{\alpha}$ (3mks)

(b) The loss function under a decision problem is given by:

	θ_1	θ_2	θ_3
d_1	14	12	13
d_2	13	15	14
d_3	11	15	5

Determine the minimax solution to this problem. (4mks)

(c) An investor is contemplating an investment with a return of R, where:

$$R = 250,000 - 100,000N$$

and N is a Normal $[1,1]$ random variable. Calculate each of the following measures of risk:

(i) Variance of return and downside semi-variance of return (5mks)

(ii) Shortfall probability, where the shortfall level is 50,000 (5mks)

Question Three (20 marks)

(a) A market trader has the option for one day of selling either ice-cream (d_1), hot food (d_2) or umbrellas (d_3) at an outdoor festival. He believes that the weather is equally likely to be fine (θ_1), overcast (θ_2) or wet (θ_3) and estimates his profits under each possible scenario to be:

	θ_1	θ_2	θ_3
d_1	25	19	7
d_2	10	30	8
d_3	0	2	34

Determine the minimax solution to this problem. (5mks)

(b) Use change of variable technique given

$$f(x) = \frac{\alpha}{x_0}(\frac{x_0}{x})^{\alpha+1} \qquad x > x_0$$

and $x = x_0 e^y$ to find the distribution $g(y)$ (6mks)

(c) List the four main perils typically covered by employer s liability insurance. (4mks)

(d) An investor is considering investing in one of two assets. The distribution of returns from each asset is shown below:

Asset 1		*Asset 2*	
Return (%)	*Probability (%)*	*Return (%)*	*Probability (%)*
−1	$8^1/_3$	0	50
11	$91^2/_3$	20	50

Calculate for each asset, the variance, semi-variance and shortfall probability where necessary assume a benchmark return of 0%. (5mks)

Question Four (20 marks)

(a) The loss function for this decision is shown below:

	d_1	d_2	d_3
θ_1	0	5	8
θ_2	12	0	3
θ_3	20	15	0

Determine the minimax solution when assigning an applicant to a category. (3mks)

(b) List the main perils insured against under a household buildings policy. (2mks)

(c) Using the exponential family distribution constructed in question 1(a) use the properties of exponential families to confirm that the mean and variance of the distribution are μ and $\frac{\mu^2}{\alpha}$. (5mks)

(d) Proof that

$$\beta(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

showing the relationship between β and Γ (5mks)

(e) Use cumulative distribution technique given

$$f(x) = \frac{\alpha}{x_0}\left(\frac{x_0}{x}\right)^{\alpha+1} \qquad x > x_0$$

and $x = x_0 e^y$ to find the distribution $g(y)$ (5mks)

Question Five (20 marks)

(a) Claim sizes (in suitable units) for a portfolio of insurance policies come from a distribution with probability density function

$$f(x) = \begin{cases} axe^{-x^2} & \text{if } 0 < x < 2, \quad a > 0 \\ 0 & \text{otherwise} \end{cases}$$

Find the value of a (5mks)

(b) The loss function under a decision problem is given by:

	θ_1	θ_2	θ_3
d_1	11	9	19
d_2	10	13	17
d_3	7	13	10
d_4	16	5	13

State which decision can be discounted immediately and why then determine the minimax solution in this case. (5mks)

(c) Prove that $\frac{G'(S)}{G(s)} = \frac{(a+b)}{1-as}$ follows Poisson distribution. (5mks)

(d) Explain different types of risk (5mks)