



KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

SPECIAL EXAMINATION

**FIRST YEAR EXAMINATION FOR THE AWARD OF
DEGREE IN BACHELOR OF SCIENCE IN MATHEMATICS & COMPUTING
FIRST SEMESTER 2021/2022
(JULY, 2022)**

MATH 111: CALCULUS 1

STREAM: Y1 S1

TIME: 2 HOURS

DAY: WEDNESDAY, 8.00 AM – 10.00 AM

DATE: 20/07/2022

INSTRUCTIONS:

- 1. Do not write anything on this question paper.***
- 2. Answer Question ONE (Compulsory) and any other TWO Questions.***

QUESTION ONE (30 marks)

- If $x = \cos t$ and $y = 1 - \sin 2t$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ (4marks)
- State three conditions for a function to be continuous at a point $x = a$ (3marks)
- Find $\frac{dy}{dx}$ if $y = e^{3x}(\sin 2x)$ (3marks)
- Find $f'(x)$ for $f(x) = x^2 + 4x - 7$ from first principle (4marks)
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- Investigate the nature of the turning point of the curve $y = 3x^2 + 6x^2 - 15x + 51$
 - Locate the turning point (5marks)
 - Determine the nature of the turning point
- Find the equation of the tangent and the normal to the curve $y = x^3 - 2x^2 + 3x - 1$ at the point $P(2, 5)$. (6marks)

- h) Evaluate the limit $\lim_{x \rightarrow 0} \frac{\sin(3x)}{\sin(8x)}$ (5marks)

QUESTION TWO (20 marks)

- a) Find $f'(x)$ given that $f(x) = \log_3 \sin(2x^2 + 5)$
(5marks)
- b) A 15m ladder is placed to rest against the wall so that the bottom is 10m away from the wall. If the ladder is pushed towards the wall at a rate of 0.25m per sec. Determine how fast is the top of the ladder moving up the wall at $t = 12 \text{ sec}$.
(7marks)
- c) find $\frac{dy}{dx}$ if $xy + \ln(x + y) = 1$
(5marks)
- d) find $\frac{dy}{dx}$ if $y = \cos(\cos x)$

(3marks)

QUESTION THREE (20 marks)

- a) Find from the first principles, the derivative of the functions
(6marks)
- $f(x) = \sin x$
 - $f(x) = \sqrt{x + 2}$
- b) Find the derivatives of the following functions using appropriate methods
- $y = \frac{x^2 + 3x - 4}{2x + 1}$ (4marks)
 - $y = \cos(5x^3)$ (3marks)
- c) Differentiate the implicit function;
- d) $x^2 + 2y^3$ (4 marks)
- e) find $\frac{dy}{dx}$ if $y = \cos(\cos x)$ (3marks)

QUESTION FOUR (20 marks)

a) Given the function $g(y) = \begin{cases} y^2 + 5 & \text{if } y < -2 \\ 1 - 3y & \text{if } y \geq -2 \end{cases}$. Compute the $\lim_{y \rightarrow -2} g(y)$

(5marks)

b) At what point does the tangent to the function $y = x^3 + 2x^2$ have a slope of zero.

(6marks)

c) Given that $P = 3q^4 - 4q^2 + 3$; $\frac{d^3p}{dq^3}$

(4marks)

d) A church window with rectangular bottom and a semi-circular top is build using a 12m framing material. Determine the dimensions of the window to let in most light.

(5marks)

QUESTION FIVE (20 marks)

e) The amount of air in a balloon at any time t is given by $V(t) = \frac{6\sqrt[3]{t}}{4t+1}$. Determine if the balloon is being drained or being filled with air at $t = 8$.

(4marks)

f) Evaluate the derivative of the function $f(t) = \frac{t}{1+t}$ from the first principles

(4marks)

g) Differentiate $y = \tan^{-1} 3x^2$

(4marks)

a) Evaluate $\lim_{x \rightarrow 0} \frac{\tan 2x}{x}$

(4marks)

i. The parametric equation of the curve are $x = e^t$ and $y = \sin t$. Find $\frac{dy}{dx} \frac{d^2y}{dx^2}$

(4marks)