KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

SPECIAL EXAMINATION

FIRST YEAR EXAMINATION FOR THE AWARD OF DEGREE IN BACHELOR OF EDUCATION SCIENCE

SECOND SEMESTER 2021/2022

(JULY, 2022)

MATH 113: VECTORS AND MECHANICS

STREAM: Y1 S2 **TIME: 2 HOURS**

DAY: TUESDAY, 3.00 PM – 5.00 PM **DATE: 26/07/2022**

INSTRUCTIONS:

1. Do not write anything on this question paper.
2. Answer Question ONE (Compulsory) and any other TWO Questions.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) If $\boldsymbol{A} = (4,\ 2,\ 5)$ and $\boldsymbol{B} = (5,\ 7, -3)$. Find $\overrightarrow{\boldsymbol{AB}}$ and $\left|\overrightarrow{\boldsymbol{AB}}\right|$. (3marks)

b) A body is projected vertically upwards with an initial velocity of $75m/s$. Find:
   i) Time taken to reach maximum height. (2marks)
   ii) The maximum height reached. (2marks)
   iii) The time of flight. (2marks)

c) Two masses $20kg$ and $8kg$ are connected to a fine string which passes over a smooth pulley fixed at the end of an inched plane which is 17 $metres$ long and $8metres$ high. The heavier particle rests on the plane and the lighter particle hangs on the pulley. Find the acceleration of the masses and the tension in the string. (5marks)

d) The sides of a square lamina A, B, C, D are each $4metres$ long. Along $AB, CB, CD, DA\ and\ BD$ act as forces of magnitude $2, 4, 6, 8\ and\ 10\ N$ respectively. Find the algebraic sum of their moments about:
   i) Point A (2marks)
   ii) The center of the square. (2marks)

e) Explain the general principle of moments. (3marks)

f) A body of mass $15kg$ is held on a smooth slope at 30^0 to the horizontal. Find the acceleration of the body when it is released and the normal reaction to the body. (5marks)

g) If $\boldsymbol{A} = 4i - 4j + 3k \quad and \quad \boldsymbol{B} = -2i + j - 2k$. Find the unit vector:
i)in the direction of **A** (2marks)
ii) perpendicular to the plane of **AB** (2marks)

QUESTION TWO (20MARKS)

a) Derive the third equation of motion under constant acceleration. (4marks)

b) A body has an initial velocity of $150m/s$ and is subjected to an acceleration of $-20m/s/s$. find its velocity after 4 $seconds$. (3marks)

c) A non-uniform ladder $30metres$ long and $50kg$ mass rests with the end A against a smooth vertical wall and the other end B on a smooth ground $6metres$ from the wall. It is maintained in this position by a horizontal cord attached at B. If the centre of gravity is $10metres$ from B; find:
i)Normal reaction at A (3marks)
ii)Tension in the cord (1mark)
iii)Normal reaction at B (2marks)

d) The distance $x\ metres$ moved by a body after t seconds is given by $x = t^2 + 4t + 3\ metres$. Find its velocity and acceleration after 3 seconds. (3marks)

e) If $\boldsymbol{A} = 2i - j + 3k\ and\ \boldsymbol{B} = -3i - 2j + 5k$. Find:
i)The magnitude of $\boldsymbol{A} + \boldsymbol{B}$ (2marks)
ii)The angle between the vectors $\boldsymbol{A}\ and\ \boldsymbol{B}$. (2marks)

QUESTION THREE (20MARKS)

a) Prove that the following vectors are parallel:
$\boldsymbol{u} = 2i - 3j + k \quad and \quad \boldsymbol{v} = -i + \frac{3}{2}j - \frac{1}{2}k$ (3 marks)

b) The mass of a vehicle is $1500kg$. The combined frictional and resistance is $750N$. Find the driving force of the vehicle, if it is traveling on a level surface with acceleration of $3m/s/s$. (3marks)

c) Find the displacement and velocity of a particle after 3 $seconds$ whose initial velocity is $\begin{pmatrix} 2 \\ 5 \end{pmatrix} m/s$ if it had a constant acceleration of $\begin{pmatrix} 3 \\ -4 \end{pmatrix} m/s/s$. (3marks)

d) A particle moving in a straight line has an acceleration of $(2t - 9)m/s/s$ after t seconds. If its velocity at $t = 0\ is\ 18m/s$. Find the value of t for which it is stationary. (5marks)

e) Four vectors of magnitude 4, $5\sqrt{2}$, $6\ and\ 8$ units inclined at angles of $45^0, 60^0, 75^0, and\ 150^0$ to the x-axis. Find the magnitude and direction of the resultant vector R. (4marks)

f) A vector $\overrightarrow{\boldsymbol{OP}}$ has a magnitude of 4 units in the direction of 30^0. Find its resolution along the $x\ and\ y - axis$ respectively. (2marks)

QUESTION FOUR (20MARKS)

a) Given the vectors $\boldsymbol{A} = 2i - 4j + 3k \; and \; \boldsymbol{B} = \; 2i - 3j + 5k$, find:
i)$\boldsymbol{A} \times \boldsymbol{B}$ ii) $\boldsymbol{B} \times \boldsymbol{A}$ (4marks)

b) Forces of magnitude $3, 7 \; and \; 5N$ act along the sides $AB, BC \; and \; CA$ of an equilateral triangle. Find the magnitude and direction of the resultant with AB along the $x - axis$. (5marks)

c) A body of mass 8kg is allowed to slide down a rough surface inclined at 40^0 to the horizontal against a frictional force of $20N$. find:

i) the normal reaction R (2marks)

ii) the co-efficient of friction (2marks)

iii) The acceleration of the body (2marks)

d) A body of mass $4kg$ is placed on a smooth surface and is connected by a string over a smooth pulley to a mass of $6kg$hanging freely. The system is released. Find the acceleration of the system and the tension on the string. (5marks)

QUESTION FIVE (20MARKS)

a) A particle of weights $3, 4, 5 \; and \; 6N$ are placed at the vertices of a rectangular frame $A, B, C \; and \; D$ respectively. If $AB = 6cm \; and \; BC = 12cm$, find the position of the centre of gravity relative to the point A. (5marks)

b) If $A = 2i - 3j + k, \; B = i + 4j - 2k \; \; and \; C = -3i + 2j + k$, find:
i) $A.B \quad ii) \; A \times C \quad iii) \; A.(B \times C)$ (6marks)

c) Show that the area of a parallelogram is given by $|\boldsymbol{A} \times \boldsymbol{B}|$. Hence, find the area of a parallelogram with sides $\boldsymbol{A} = 3i + j - 2k, \; \boldsymbol{B} = i - 3j + 4k$. (5marks)

d) The path of a particle after t seconds is given by $r = ti + (3t - t^3)j$. Find its velocity and acceleration after 3 seconds. (4marks)