KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

SPECIAL EXAMINATION

FIRST YEAR EXAMINATION FOR THE AWARD OF DEGREE IN BACHELOR OF SCIENCE IN APPLIED STATISTICS

SECOND SEMESTER 2021/2022

(JULY, 2022)

MATH 116: MATRIX ALGEBRA

STREAM: Y1 S2 **TIME: 2 HOURS**

DAY: TUESDAY, 11.30 AM – 1.30 PM **DATE: 26/07/2022**

INSTRUCTIONS:

1. Do not write anything on this question paper.

2. Answer Question ONE (Compulsory) and any other TWO Questions.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Calculate the determinant of the following matrices:

i) $A = \begin{pmatrix} 3 & 5 \\ 4 & -3 \end{pmatrix}$ ii) $A = \begin{bmatrix} 4 & -7 & 6 \\ -2 & 4 & 0 \\ 5 & 7 & -4 \end{bmatrix}$ (6marks)

b) Determine the inverse of the following matrices:

i) $A = \begin{pmatrix} \frac{1}{2} & \frac{2}{3} \\ \frac{1}{3} & -\frac{3}{5} \end{pmatrix}$ ii) $A = \begin{bmatrix} 1 & 5 & -2 \\ 3 & -1 & 4 \\ -3 & 6 & -7 \end{bmatrix}$ (7marks)

c) Use matrices to solve:

i) $3x + 5y - 7 = 0$

$4x - 3y - 19 = 0$ (3marks)

ii) $3a + 4b - 3c = 2$

$$-2a + 2b + 2c = 15$$

$$7a - 5b + 4c = 26$$ (5marks)

d) A circuit comprises of three loops. Applying Kirchoff's laws to the closed loops gives the following equations for current flow in mA.

$$2I_1 + 3I_2 - 4I_3 = 26$$
$$I_1 - 5I_2 - 3I_3 = -87$$
$$-7I_1 + 2I_2 + 6I_3 = 12$$

(5marks)

e) Use Cramer's rule to solve:

$$5T_1 + 5T_2 + 5T_3 = 7.0$$
$$T_1 + 2T_2 + 4T_3 = 2.4$$
$$4T_1 + 2T_2 = 4.0$$

(4marks)

QUESTION TWO (20MARKS)

a) Use Gauss-Elimination method to solve:

$$10x - 2y - 3z = 205$$
$$2x - 10y + 2z = -154$$
$$2x + y - 10z = -120$$

(6marks)

b) Find the eigenvalues of the matrix:

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$ (7marks)

c) State and prove the Cayley-Hamilton Theorem. (7marks)

QUESTION THREE (20MARKS)

a) Find the characteristic polynomial of the matrix

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

Hence find A^{-1}. (7marks)

b) Show that the vector $(1, 1, 2)$ is an eigenvector of the matrix $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$ corresponding to eigenvalue 2. (3marks)

c) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} -2 & 5 & 4 \\ 5 & 7 & 5 \\ 4 & 5 & -2 \end{bmatrix}$$ (10marks)

QUESTION FOUR (20MARKS)

Find the modal matrix P and the resulting diagonal matrix D of A, if:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$ (20marks)

QUESTION FIVE (20MARKS)

a) Reduce quadratic form to canonical form using orthogonal transformation. Also find the nature, index and signature of the resulting matrix.

$$10x_1^2 + 2x_2^2 + 5x_3^2 + 6x_2x_3 - 10x_3x_1 - 4x_1x_2$$

(14marks)

b) The relationship between the displacement, s, velocity, v, and acceleration, a, of a piston is given by the equations:

$$s + 2v + 2a = 4$$
$$3s - v + 4a = 25$$
$$3s + 2v - a = -4$$

Use matrices to determine the values of s, v and a. (6marks)