KISII UNIVERSITY

THIRD YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF BACHELOR OF SCIENCE

JANUARY- APRIL, 2020

MATH 191: ENGINEERING MATHEMATICS II

STREAM: Y2S2 **TIME: 2 HOURS**

Instructions

- Question ***ONE*** is compulsory
- Attempt any other two questions from the remaining four questions.
- Question 1 carries **30 marks** while each of the others carries **20 marks**

QUESTION ONE

a) Resolve the following into partial fractions:

$$\frac{6x-5}{(x-1)(x^2+3)}$$

(3 marks)

b) Given the system of equations:

$$-3x_1 + 7x_3 = 4$$
$$x_1 + 2x_2 - x_3 = 0$$
$$5x_1 - 2x_2 \quad = 3$$

Use Cramer's rule to solve the system. **(3 marks)**

c) Find the first 6 terms of the Taylor series for $\ln x$ about the point $x = 2$. **(4 marks)**

d) From Einstein's theory of special relativity, it is known that the mass of an object is a function of its speed v. This relationship is given by

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

By considering the case when $\frac{v^2}{c^2} \ll 1$, provide a binomial power series approximation for the mass m of the object. **(3 marks)**

e) Use Maclaurin's power series to evaluate $\frac{1}{4}(x^2-1)xe^{2x}$ as far as the term in x^7. **(4 marks)**

f) **(a)** Find the projection of the vectors $\vec{\boldsymbol{P}} = 3\boldsymbol{i} - \boldsymbol{j} + 5\boldsymbol{k}$ and $\vec{\boldsymbol{Q}} = -6\boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k}$. **(2 marks)**

(b) If the vectors $\vec{\boldsymbol{a}} = 2\boldsymbol{i} - 3\boldsymbol{j}$ and $\vec{\boldsymbol{b}} = -6\boldsymbol{i} + m\boldsymbol{j}$ are collinear, find the value of m. **(2 marks)**

g) Evaluate $\ell im_{x \to \infty}\left(\frac{1-\cos x}{x^2}\right)$ **(3 marks)**

h) Use De Movre's theorem to evaluate z^5 where $z = -2 + i\sqrt{5}$. **(2 marks)**

i) Evaluate $\int_0^{\frac{\pi}{2}} 3x^2 Cos(\frac{1}{2}x)\, dx$, using Bernoulle's integral method. **(4 marks)**

QUESTION TWO

a) Given the following function, find y'''.

$$y(x) = \frac{x}{b}\sqrt{(a-x)x^2}$$

(5 marks)

b) Find y'' if $y = \frac{1}{2}\tan(4x)$ **(5 marks)**

c) Given $z_1 = -2 + i\sqrt{3}$ and $z_2 = 3 - i\sqrt{3}$, determine $|z_1 - z_2|^2$ **(2 marks)**

d) Resolve the following into partial fractions:

$$\frac{3x^2 + 16x + 15}{(x+3)^3}$$

(3 marks)

e) Find the eigen values that satisfy the following matrix equation.**(3marks)**

$$\begin{vmatrix} (2-\lambda) & -2 \\ 1 & (5-\lambda) \end{vmatrix} = 0$$

f) Use integration by parts to evaluate the integral:

$$\int_1^2 \frac{1}{5}(\ln x)x^3 \ dx$$ **(2 marks)**

QUESTION THREE

a) Given that z is a complex number, determine the polar form of

$$z = ln(3-7i)$$

(3 marks)

b) The forces in three members of a framework are $F_1, F_2,$ and F_3. They are related by the following simultaneous equations:

$$1.4F_1 + 1.8F_2 + 2.8F_3 = 5.6$$
$$4.2F_1 - 1.4F_2 + 5.6F_3 = 35.0$$
$$4.2F_1 - 2.8F_2 - 1.4F_3 = -5.6$$

Find the values of $F_1, F_2,$ and F_3 using Gaussian elimination. **(5 marks)**

c) Resolve the following into partial fractions then evaluate the integral

$$\int_2^3 \frac{x^3 - 2x^2 - 4x - 4}{x^2 + x - 2} dx$$

(5 marks)

d) **(i)** Prove that the vectors $\vec{\boldsymbol{W}} = \boldsymbol{3}i - \boldsymbol{j} + 5\boldsymbol{k}$ and $\vec{\boldsymbol{U}} = \boldsymbol{3}i - \boldsymbol{j} + 5\boldsymbol{k}$ are perpendicular. **(2 marks)**

(ii) Given the vectors $\vec{\boldsymbol{A}} = i + 2\boldsymbol{j} - 5\boldsymbol{k}$ and $\vec{\boldsymbol{B}} = \boldsymbol{2}i + \boldsymbol{j} - 2\boldsymbol{k}$, find $\vec{\boldsymbol{B}} \times \vec{\boldsymbol{A}}$. **(2 marks)**

e) Use Taylor theorem to expand $sin(x+a)$ in ascending powers of x as far as the term with x^4. Hence determine $sin(43^0)$. **(3 marks)**

QUESTION FOUR

a) Find the roots of $(5+3i)^{\frac{1}{2}}$ to 4 significant figures **(3 marks)**

b) Evaluate $\iint_1^2 4(x^3 - x^{-2})dx$ **(4 marks)**

c) The velocity v of point P on a body with angular velocity ω about a fixed axis is given by

$$\vec{\boldsymbol{v}} = \vec{\boldsymbol{\omega}} \times \vec{\boldsymbol{r}}$$

Where r is point on vector $\vec{\boldsymbol{P}}$. Find $\vec{\boldsymbol{v}}$ given that at point P, $\vec{\boldsymbol{\omega}} = \boldsymbol{2i} - \boldsymbol{5j} + \boldsymbol{7k}$ and $\vec{\boldsymbol{r}} = \boldsymbol{j} + \boldsymbol{3k}$, where $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ are unit vectors. **(4 marks)**

d) The following system of equations is designed to determine concentrations (the $c's$ in g/m^3) in a series of coupled reactors as a function of the amount of the mass input to each reactor (the right-hand sides in g/day).

$$15c_1 - 3c_2 \quad - c_3 = 4000$$
$$-3c_1 + 18c_2 \; - 6c_3 = 100$$
$$-4c_1 - \; c_2 \; + 12c_3 = 2350$$

(i) Determine the matrix inverse **(2 marks)**

(ii) Use the inverse to determine the solution of the system to determine the solution for system **(3 marks)**

e) Find the maximum value of each of the following curves:

(i) $y = 4x - 2x^2$ **(2 marks)**

(ii) $y = \frac{1}{x}(ln\, x)$ **(2 marks)**

QUESTION FIVE

a) Change $z = 3e^{2-3i}$ into the Cartesian form, z is a complex number. **(2 marks)**

b) When the displaced electrons oscillate about an equilibrium position, the displacement x is given by the equation

$$x = A\, exp\left[\frac{-ht}{2m} + i\frac{\sqrt{(4mf - h^2)}}{2ma}\, t\right]$$

Determine the real part of x in terms of t assuming $(4mf - h^2)$ is positive. **(3 marks)**

c) Given $\vec{p} = 6\boldsymbol{i} + 4\boldsymbol{k}$, $\vec{q} = 8\boldsymbol{i} - 4\boldsymbol{j} + 6\boldsymbol{k}$ and $\vec{r} = 6\boldsymbol{i} + 10\boldsymbol{j} - 8\boldsymbol{k}$, determine:

(i) $-\vec{p} + 3|\vec{r}|$ **(2 marks)**

(ii) $(\vec{q} - 3\vec{p}).\frac{1}{2}\vec{r}$ **(2 marks)**

(iii) $(\vec{r} + \vec{p}) \times \frac{1}{2}\vec{q}$ **(2 marks)**

d) Find the eigen values λ that satisfy the following equation

$$\begin{vmatrix} (5-\lambda) & 7 & -5 \\ 0 & 4-\lambda & -1 \\ 2 & 8 & -3-\lambda \end{vmatrix} = 0$$

(3 marks)

f) The tensions $T_1, T_2,$ and T_3 in a simple framework are given by the following equations:

$$5F_1 + 5F_2 + 5F_3 = 7.0$$
$$F_1 + 2F_2 + 4F_3 = 2.4$$
$$4F_1 + 2F_2 = 4.0$$

Use Gaussian elimination to find the values of $T_1, T_2,$ and T_3. **(3 marks)**

e) Use integration by parts to evaluate the integral: **(3 marks)**

$$\int x(ln\; x)\, dx$$