KISII UNIVERSITY
SCHOOL OF PURE AND APPLIED SCIENCES
BACHELOR OF MATHEMATICS
COURSE TITLE:ACTUARIAL MATHEMATICS I
COURSE CODE: BACS 212
FINAL EXAM AUGUST 2020
INSTRUCTIONS:Answer question one and any other 2 questions in section B

SECTION A (30 marks)

Question One (30 marks)

(a) An investigation was carried out into the mortality of male undergraduate students at a large university. The resulting crude rates were graduated graphically. The following table shows the observed numbers of deaths at each age x, d_x , and the q_x^0 obtained from the graduation, together with the number of lives exposed to risk at each age.

Age x	d_x	$\mathring{q}_x$	*Exposed-to-risk*
18	6	0.0012	5,200
19	8	0.0013	5,000
20	12	0.0015	4,800
21	8	0.0017	5,000
22	9	0.0019	3,800
23	6	0.0020	3,600
24	8	0.0021	3,200

Test whether the overall fit of the graduated rates to the crude data is satisfactory using a chi-squared test. (5mks)

(b) Estimate the value of ${}_{\infty}q_{74^1:84}$ on the basis of A1967-70 ultimate mortality. (Assume $l_{108} = 0$, so that the integral is over a range of 24 years. (5mks)

(c) A life office issues a 3 year unit-linked endowment policy to a life aged exactly 60. The annual premium is \$2000, payable at the start of each year. The allocation proportion is 90% in year 1 and 97% thereafter. At the end of year of death during the term, the policy pays the higher of \$10000 and the bid value of units allocated to the policy,after deduction of the fund management charge. A bonus of 2% of the (bid) value of the unit fund is payable at maturity. The life office makes the following assumptions in projecting future cashflows:

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Mortality	A1967-70 ultimate
Initial expenses:	£300
Renewal expenses:	£50, incurred at the start of the second and the third years
Fund management charge:	2% per annum, taken at the end of each year before payment of any benefits
Sterling fund interest rate:	4% per annum
Bid/offer spread:	6%
Unit fund growth rate:	10% per annum.

Construct tables to show the following:

(i) the growth of the unit fund;

(ii) the profit signature, assuming that no sterling reserves are held;

(iii) the profit signature after taking into account sterling reserves, given that the sterling reserves per policy are to be £36.48 before receipt of the premium due at time 1 year and £78.64 before receipt of the premium due at time 2 years.

In each case, indicate clearly how you calculate your table entries. Ignore the possibility of surrenders. (7mks)

(d) In the context of profit-testing, explain the difference between the profit vector and the profit signature. (3mks)

(e) A life assurance company carried out an investigation of the mortality of male life assurance policyholders. The investigation followed a group of 100 policyholders from their 60th birthday until their 65th birthday, or until they died or cancelled their policy (whichever event occurred first).

(i) Calculate the Nelson-Aalen estimate of the integrated hazard. (5mks)

(ii) Explain types of censoring and sketch the integrated hazard function (5mks)

The ages at which policyholders died or cancelled their policies were as follows:

Died	**Cancelled policy**
Age in years and months	*Age in years and months*
60y 5m	60y 2m
61y 1m	60y 3m
62y 6m	60y 8m
63y 0m	61y 0m
63y 0m	61y 0m
63y 8m	61y 0m
64y 3m	61y 5m
	62y 2m
	62y 9m
	63y 9m
	64y 5m

SECTION B

Question Two (20 marks)

(a) An investigation into the mortality of patients following a specific type of major operation was undertaken. A sample of 10 patients was followed from the date of the operation until either they died, or they left the hospital where the operation was carried out, or a period of 30 days had elapsed (whichever of these events occurred first).

Patient number	*Duration of observation (days)*	*Reason for observation ceasing*
1	2	Died
2	6	Died
3	12	Died
4	20	Left hospital
5	24	Left hospital
6	27	Died
7	30	Study ended
8	30	Study ended
9	30	Study ended
10	30	Study ended

(i) Calculate the value of the Kaplan-Meier estimate of the survival function at duration 28 days. (5mks)

(ii) Write down the Kaplan-Meier estimate of the hazard of death at duration 8 days. (2mks)

(iii) Sketch the Kaplan-Meier estimate of the survival function. (3mks)

(b) An airline runs a frequent flyer scheme with four classes of member: in ascending order Ordinary, Bronze, Silver and Gold. Members receive benefits according to their class. Members who book two or more flights in a given calendar year move up one class for the following year (or remain Gold members), members who book exactly one flight in a given calendar year stay at the same class, and members who book no flights in a given calendar year move down one class (or remain Ordinary members). Let the proportions of members booking $0, 1$ and $2+$ flights in a given year be p_0, p_1 and p_2+ respectively.

(i) Explain how this scheme can be modelled as a Markov chain and why there must be a unique stationary distribution for the proportion of members in each class. (2mks)

(ii) Write down the transition matrix of the process. (3mks)

(iii) The airline's research has shown that in any given year, 40% of members book no flights, 40% book exactly one flight, and 20% book two or more flights. Calculate the stationary probability distribution. (5mks)

Question Three (20 marks)

(a) A life office issues 3-year term assurance policy to a man aged exactly 59. The sum assured is 15;000, payable at the end of the year of death. Level premiums are payable annually in advance. Expenses are expected to be as follows:
initial expenses: 10 , renewal expenses: 2 incurred at the beginning of the 2nd and each subsequent policy year. It is assumed that interest of 7% per annum will be earned on the life funds, and that mortality follows the A1967-70 ultimate table. The risk discount rate used by the o±ce is 15% per annum. The office calculates the annual premium by requiring that the net present value of the expected profit on each policy is equal to 20% of one office premium. Calculate the office premium on each of the following reserving bases:

(i) The office holds zero reserves at each year-end. (5mks)

(ii) The office sets up a reserve at each year-end (except the last) equal to 80% of one office premium. (5mks)

(b) The staff of a large company is maintained as a stationary population by 500 new entrants each year at exact age 20. One third of those reaching age 30 leave immediately. Of the remainder, $\frac{1}{4}$ of those attaining age 60 retire immediately and the survivors retire at age 65. The only other decrement is death. Calculate

(i) The number of staff. (2mks)

(ii) The number of deaths in service each year. (2mks)

(c) Mr Bunn the baker made 12 pies to sell in his shop. He placed the pies in the shop at 9 a.m. During the rest of the day the following events took place. Calculate

Time	*Event*
10 a.m.	A boy bought two pies
11 a.m.	A man bought three pies
12 noon	Mr Bunn accidentally sat on one pie and squashed it so it could not be sold
1 p.m.	A woman bought two pies
2 p.m.	A dog from across the street ran into Mr Bunn's shop and stole two pies
3 p.m.	A girl on the way home from school bought one pie
5 p.m.	Mr Bunn closed for the day and the remaining pie was still in the shop

(i) Estimate the time it takes Mr Bunn to sell 40% of the pies he makes, using the Nelson-Aalen estimator. (6mks)

Question Four (20 marks)

(a) You want to use Cox regression to estimate the force of mortality for a group of endowment assurance policyholders. You start by investigating the model:

$$\mu(x, z_1, z_2) = \mu_0(x)\, e^{\beta_1 Z_1 + \beta_2 Z_2}$$

where x denotes the age of the policyholder,

$$Z_1 = \begin{cases} 0 & \text{if the duration is less than 1 year} \\ 1 & \text{if the duration is at least 1 year} \end{cases}$$

$$Z_2 = \begin{cases} 0 & \text{for males} \\ 1 & \text{for females} \end{cases}$$

You have estimated the values of the parameters β_1 and β_2, and have obtained the following results:

Covariate	Parameter	Standard error
Duration	0.416	0.067
Sex	–0.030	0.017

(i) State the class of policyholders to which the baseline hazard refers. (2mks)

(ii) Explain why duration covariate is significant in determining mortality. (3mks)

(iii) Compare the force of mortality for a new female policyholder to that of a male policyholder of the same age, who took out a policy 2 years ago. (5mks)

(b) Estimate ${}_2p_{63.25}$ assuming ELT15 (Males) mortality at integer ages and:

(i) a uniform distribution of deaths between integer ages (5mks)

(ii) a constant force of mortality between integer ages. (5mks)

Question Five (20 marks)

(a) A simple NCD system has four levels of discount $0\%, 20\%, 40\%$ and 60%. A new policyholder starts on 0% discount. At the end of each policy year, policyholders will change levels according to the following rules:

. At the end of a claim free year, a policyholder moves up one level, or remains on the maximum discount.

. At the end of a year in which exactly one claim was made, a policyholder drops back one level, or remains at 0%.

. At the end of a year in which more than one claim was made, a policyholder drops back to zero discount.

For a particular policyholder in any year, the probability of a claim free year is $\frac{7}{10}$, the probability of exactly one claim is $\frac{1}{5}$ and the probability of more than one claim is $\frac{1}{10}$.

(i) Determine transition matrix for this time homogeneous Markov chain. (3mks)

(ii) Calculate the 2-step transition probabilities from state i to state j ,p_{ij}^{2} .(3mks)

(iii) If the policyholder starts with no discount, calculate the probability that this policyholder is at the maximum discount level 5 years later. (7mks)

(iv) If a large number of people having the same claim probabilities take out policies at the same time, calculate the proportion would you expect to be in each discount category in the long run. (7mks)