KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

SPECIAL EXAMINATION

SECOND YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE EDUCATION

FIRST SEMESTER 2021/2022

(JULY, 2022)

MATH 206: INTRODUCTION TO REAL ANALYSIS

STREAM: Y2 S1 **TIME: 2 HOURS**

DAY: MONDAY, 3.00 PM – 5.00 PM **DATE: 25/07/2022**

INSTRUCTIONS:

1. Do not write anything on this question paper.
2. Answer question ONE (Compulsory) and any other TWO Questions

QUESITION ONE

a) Prove that between any two rational numbers there is another rational number (3marks)

b) Prove that for any positive real number x, there always exists a non-negative real number x^{-1} (3marks)

c) Given a subset s of $\mathbb{R}$, distinguish between the upper boundary and the supremum of s (3marks)

d) Define the following classification of the real number system giving examples
   i) Natural numbers
   ii) Integers
   iii) Rational numbers
   iv) Irrational numbers (4marks)

e) Show that if x and y are positive real numbers, then $x + y$ is also positive (4marks)

f) Prove that a subset $(2,5)$ is open in $\mathbb{R}$ (4marks)

g) Distinguish between increasing and decreasing sequences (4marks)

h) Show that the function $f(x) = x$ is Riemann integrable over [0,1]and $R\int_0^1 f = \frac{1}{2}$ (5marks)

QUESTION TWO

a) Show that $\sqrt{3}$ is not a rational number (5marks)

b) Let $m \in \mathbb{Z}$, show that m is even if and only if m^2 is even (6marks)

c) Prove that if a limit of a sequence exists, then it is unique (5marks)

d) Let S be a non-empty subset of $\mathbb{R}$. Show that the real number M is the supremum of S if and only if both the following conditions are satisfied

i)$x \leq M \; \forall \; x \in S$

ii) $\forall \; \epsilon > 0, \exists \; x' \in S : M - \epsilon \; < x' \; \leq M$ (4marks)

QUESTION THREE

a) In each of the following subsets of $\mathbb{R}$, determine if possible the upper boundary, lower boundary, upper and lower bounds

i) $S = \{x \in \mathbb{R} : 0 \leq x < 1\}$,

ii) $S = \{x \in \mathbb{R} : 2 \leq x < 5\}$,

iii) $S = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$ (6marks)

b) Let (x_n) be a sequence of real numbers. Prove that if (x_n) is convergent then it is Cauchy (4marks)

c) Show that if the limit of a function $f(x)$ exists as $x \to x_0$, then that limit point is unique (5marks)

d) Show that the function $f(x) = x^{\frac{1}{3}}$ is continuous at $x = 0$ but is not differentiable at this point

e) Define a countable set. Hence illustrate that the set of real numbers $\mathbb{R}$ is uncountable

(5marks)

QUESTION FOUR

a) Find the derivatives of the following functions from first principle

i) $y = 2x - 3$

ii) $y = x^3 + 2x$ (6marks)

b) Using the first principle, prove that

i) $\lim_{n\to\infty}\left(\frac{5n+7}{8n+9}\right) = \frac{5}{8}$

3marks

ii) $\lim_{x\to\infty}\frac{1}{x^3+1} = 0$ 3marks

iii) $\lim_{x\to\infty}(x + \sin x) = \infty$ 3marks

c) In each of the following sequences, determine the lim sup and lim Inf where possible

i) $(x_n) = (-1)^n : n \in \mathbb{N}$

ii) $(x_n) = n(1 + (-1)^n)$ (5marks)