KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

SPECIAL EXAMINATION

SECOND YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS /EDUCATION SCIENCE/ARTS

FIRST SEMESTER 2021/2022

(JULY, 2022)

MATH 220: VECTOR ANALYSIS

STREAM: Y2 S2 **TIME: 2 HOURS**

DAY: FRIDAY, 3.00 PM – 5.00 PM **DATE: 29/07/2022**

INSTRUCTIONS:

1. Do not write anything on this question paper.
2. Answer ALL Questions in SECTION A (Compulsory) and any other TWO Questions in section B.

SECTION A (30 MARKS)

1.

a. Interpret the chemical equation $2NH_2 + H_2 = 2NH_3$ as a relation in the algebra of ordered pairs. (5 marks)

b. Verify Green's theorem for $P(x, y) = x$ and $Q(x, y) = xy$, where D is the unit disc $x^2 + y^2 \leq 1$. (5 marks)

c. Verify the Cauchy–Schwarz inequality for $\mathbf{a} = -2\mathbf{i} + \mathbf{j} + 5\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + 4\mathbf{k}$. (5 marks)

d. Find the area of the triangle with vertices at the points (1, 1), (0, 2), and (3, 2) (5 marks)

e. Find unit vectors **a**, **b**, and **c** in the plane such that **b** + **c** = **a**. (2 marks)

f. Evaluate the determinant of

$$\begin{vmatrix} 36 & 18 & 17 \\ 45 & 24 & 20 \\ 3 & 5 & -2 \end{vmatrix}$$ (5 marks)

g. Find the Cartesian coordinates of the spherical coordinate point (3, $\pi/6$, $\pi/4$) and plot. (3 marks)

SECTION B (40 MARKS)

2.

a. A bird is flying in a straight line with velocity vector 10**i** + 6**j** + **k** (in kilometers per hour). Suppose that (x, y) are its coordinates on the ground and z is its height above the ground. (6 marks)

   i. If the bird is at position (1, 2, 3) at a certain moment, what is its location 1 hour later? 1 minute later?

   ii. How many seconds does it take the bird to climb 10 meters?Use the method of partial fractions to integrate

b. Let **F** = *ye z* **i** + *xez***j** + *xyez***k**. Show that the integral of **F** around an oriented simple closed curve C that is the boundary of a surface S is 0 using Stokes Theorem. (Assume S is the graph of a function)

(4 marks)

c. Find the volume of the parallelepiped spanned by the three vectors **i** + 3**k**, 2**i** +**j**−2**k**, and 5**i** + 4**k**. (4 marks)

d. Let a point have Cartesian coordinates (2, −3, 6). Find its spherical coordinates and plot. (3 marks)

e. Find an equation for the plane containing the three points (1, 1, 1), (2, 0, 0), and (1, 1, 0). (3 marks)

3.

a. Express the surface $xz = 1$ and the surface $x_2 + y_2 - z_2 = 1$ in spherical coordinates (5 marks)

b. Show that the vectors $\mathbf{i}_\theta = (\cos\theta)\mathbf{i} + (\sin\theta)\mathbf{j}$ and $\mathbf{j}_\theta = -(\sin\theta)\mathbf{i} + (\cos\theta)\mathbf{j}$ are orthogonal. (3 marks)

c. Suppose $\mathbf{a} = \alpha\mathbf{b} + \beta\mathbf{c}$; that is, $\mathbf{a} = (a_1, a_2, a_3) = \alpha\,(b_1, b_2, b_3) + \beta\,(c_1, c_2, c_3)$. Show that

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$$ (5 marks)

d. Let a point have spherical coordinates $(1, -\pi/2, \pi/4)$. Find its Cartesian coordinates and plot. (5 marks)

e. If $\mathbf{a} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$, calculate $2\mathbf{a} \cdot \mathbf{b}$.(2 marks)

4.

a. Use Stokes' theorem to evaluate the line integral $\int_C (-y^3 dx + x^3 dy - z^3 dz)$ (5 marks)

b. If a point has cylindrical coordinates $(8, 2\pi/3, -3)$, plot and find its Cartesian coordinates. (5 marks)

c. Find the spherical coordinates of the Cartesian point $(1, -1, 1)$ and plot.

(5marks)

d. Find $(3i - j + k) \times (i + 2j - k)$. (3 marks)

e. Find the components of the vector from $(3, 5)$ to $(4, 7)$ (2 marks)

5.

a. Find the equations of the line in space through the point $(3, -1, 2)$ in the direction $2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$. (2 marks)

b. Find the equation of the line in the plane through the point $(1, -6)$ in the direction of $5\mathbf{i} - \pi\mathbf{j}$. (2 marks)

c. In what direction does the line $x = -3t + 2$, $y = -2(t - 1)$, $z = 8t + 2$ point? (2 marks)

d. Find the equation of the line through $(2, 1, -3)$ and $(6, -1, -5)$. (2 marks)

e. Calculate $(\mathbf{i} + 3\mathbf{j} - \mathbf{k}) \cdot (3\mathbf{k} - 2\mathbf{j})$. (2 marks)

f. Let $\mathbf{F} = (xy^2, y + x)$. Integrate $(\nabla \times \mathbf{F}) \cdot \mathbf{k}$ over the region in the first quadrant bounded by the curves $y = x^2$ and $y = x$. (10 marks)