KISII UNIVERSITY
MAIN CAMPUS
UNIVERSITY EXAMINATIONS
THIRD YEAR EXAMINATIONS FOR THE AWARD OF THE DEGREE OF BACHELOR OF ACTUARIAL SCIENCE
SECOND SEMESTER 2019/2020
(JANUARY-APRIL 2020)

BACS 306: STOCHASTIC PROCESSES FOR ACTUARIAL SCIENCE

STREAM: BACS Y3S2 **TIME**: 2 HOURS
DAY: **DATE**:

INSTRUCTIONS:

1. ***Do not write anything on this question paper.***
2. ***Answer question ONE and any other TWO questions.***

Question One (Compulsory) (30 Marks)

(a) Explain the concept of random phenomena and describe two examples together with their mathematical modeling by stochastic processes in Actuarial Science. (4 marks)

(b) Given a sequence of cubes, $0, 1, 8, \cdots, n^3$, show that the resulting generating function is given by

$$A(s) = \frac{s(s^2 + 4s + s)}{(1-s)^4}$$

(6 marks)

(c) Suppose the stock price of a company is observed at random points in time. Let X_n denote the time (in months) that the n^{th} stock price is observed. Describe fully the stochastic process $X_n, n \geq 1$. (3 marks)

(d) A stochastic process is defined by

$$X(t) = T + (1 - t)$$

where T is a uniform random variable in $(0, 1)$. Determine the

(i) Mean, $\mu_X(t)$ of $X(t)$. (5 marks)

(ii) Autocorrelation, $\rho_X(t_1, t_2)$ of $X(t)$. (6 marks)

(iii) Autocovariance, $|\gamma_X(t_1, t_2)$ of $X(t)$. (6 marks)

Question Two (20 Marks)

(a) Consider a Markov chain with three states, 1, 2 and 3, and transition matrix

$$\begin{pmatrix} \frac{2}{5} & \frac{2}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{3}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{2}{5} & \frac{2}{5} \end{pmatrix}$$

(i) Explain what is meant by the statement that a Markov chain is an irreducible recurrent chain, and show, stating any general results that you assume, that this statement is true for the present chain. (4 marks)

(ii) Find the stationary distribution for this chain. (6 marks)

(b) Let $\{X_n, n \geq 0\}$ be a sequence of iid r.v.'s with mean 0 and variance 1. Show that $\{X_n, n \geq 0\}$ is a wide-sense stationary (WSS) process and hence it must also be covariance stationary. (10 marks)

Question Three (20 Marks)

(a) State the two definitions of a counting Poisson process $\{N(t) : t \geq 0\}$ having intensity λ hence prove that the two definitions are equivalent. (16 marks)

(b) Payments of premiums arrive at an insurance firm according to a Poisson process at a rate of 0.5 per day.

(i) Find the probability that the insurance firm receives more than two payments in a day. (2 marks)

(ii) If there are more than 4 days between payments, all the paper will be used up and the presses will be idle. What is the probability that this will happen? (2 marks)

Question Four (20 Marks)

(a) Let X be a random variable having a Poisson distribution

$$p(x) = \frac{\lambda^x}{x!} e^{-\lambda}$$

Suppose $Y = bX + c$, where b and c are constants. Find the probability generating function of Y. Hence obtain the mean and variance of Y. (12 marks)

(b) Let $Z_1, Z_2, \cdots$ be independent identically distributed random variables with $P(Z_n = 1) = p$ and $P(Z_n = -1) = q = 1 - p$ for all n. Let

$$X_n = \sum_{i=1}^{n} Z_i, \quad n = 1, 2, \cdots$$

and $X_0, = 0$.

(i) Describe the simple random walk $X(n)$. (4 marks)

(ii) Now suppose $\{X_n, n \geq 0\}$, in (i) is a simple random walk defined by

$$X(t) = X_n \quad n \leq t < n + 1$$

Describe X(t). (4 marks)