



**KISII UNIVERSITY**

**UNIVERSITY EXAMINATIONS**

**SPECIAL EXAMINATION**

**THIRD YEAR EXAMINATION FOR THE AWARD OF DEGREE OF**

**BACHELOR OF SCIENCE ACTUARIAL SCIENCE**

**SECOND SEMESTER 2021/2022**

**(JULY, 2022)**

**BACS 310: ACTUARIAL MATHEMATICS II**

**STREAM: Y3 S2**

**TIME: 2 HOURS**

**DAY: TUESDAY, 8.00 AM – 10.00 AM**

**DATE: 26/07/2022**

**INSTRUCTIONS:**

- 1. Do not write anything on this question paper.***
- 2. Answer Question ONE (Compulsory) and any other TWO Questions***

**Question one (30 marks)**

1. Give a mathematical definition of the Markov property. **(3marks)**
2. Define a multiple decrement life-table giving 5 examples where it is applied insurance. **(7marks)**
3. Let  $\beta_1, \beta_2$  be the modes of decrement in a double-decrement table. Suppose that  $\beta_1$  is uniformly distributed over the year of age from  $x$  to  $x + 1$  in its associated single-decrement table, and  $\mu_{x+t}^{\beta_2} = c$  for  $0 < t < 1$  Find formulae for  $aq_x^{\beta_1}$  and  $aq_x^{\beta_2}$  in terms of  $q_x^{\beta_1}$  and  $q_x^{\beta_2}$  **(7marks)**
4. Derive from first principles the Kolmogorov forward differential equation: **(7marks)**

$$\frac{d}{dt}tp_x = -tp_x\mu_{x+t}$$

Hence show that:

$$tp_x = \exp\left(-\int_0^t \mu_{x+s} ds\right)$$

5. Define the following in reference to valuing sickness policies **(6marks)**

- (i) The deferred period
- (ii) The waiting period
- (iii) The off-period

**Question 2 (20marks)**

1. A large industrial company recruits a constant number of school leavers aged exactly 18 year on 1 July each year. Upon joining, workers undergo training for one year. Of those who complete this period of training, ten per cent fail a final test of competence and are dismissed. Employees may also leave service voluntarily at any time. The central rate of voluntary withdrawal from service is 0.15 for trainees and 0.10 at each age for fully trained employees. The occupation is hazardous and all workers, including trainees, are exposed to the risk of injury. The independent q-type rate of injury is 0.051219 at age 18 and 0.050030 at ages 19 and above. An employee who is injured is transferred to alternative work with a subsidiary company, at a relocation cost of \$1,000. The mortality of all employees follows English Life Tables No.12 - Males. The number of employees attaining age 21 each year is 500.
  - (i) Construct a service table covering the first 3 years of employment with the original company, distinguishing between those about to take the final test of competence and those who pass it. (Regard failing the test as a special mode of decrement).
  - (ii) How many people are recruited on each 1st July? **(10marks)**
2. Under UD of D assumption between ages  $x$  and  $x+1$  in each mode of decrement in a single-decrement table. Proof **(10marks)**

- (i)  $aq_x^{\beta_1} = q_x^{\beta_1} (1 - 1/2q_x^{\beta_2})$
- (ii)  $(a\mu)_x = \mu_x^{\beta_1} + \mu_x^{\beta_2}$
- (iii)  $am_x^{\beta_1} \cong \mu_{x+1/2}^{\beta_1}$
- (iv)  $aq_x^{\beta_1} = q_x^{\beta_1} [1 - \frac{1}{2}(q_x^{\beta_2} + q_x^{\beta_3}) + \frac{1}{3}q_x^{\beta_2} \cdot q_x^{\beta_3}]$

**Question 3 (20marks)**

1. A certain variety of tomato is susceptible to blight, which is always fatal. A researcher decides to model the life cycle of the tomato using a multiple state model with the following states:
  1. Not suffering from blight
  2. Suffering from blight
  3. Dead.
 The transition rates are dependent on the age of the plant and are as follows:  
 $\mu_x$  is the mortality rate at exact age  $x$  of a blight-free plant

$\sigma_x$  is the rate of contracting blight at exact age  $x$

$\tau_x$  is the mortality rate at exact age  $x$  of a plant suffering from blight.

(i) Draw and label a transition diagram for this multiple state model.

Let  $p_{ij}(x, y)$  denote the probability that a plant is in State  $j$  at age  $y$  ( $y > x$ ) given that it was in State  $i$  at age  $x$ .

(ii) Write down an expression involving transition rates for each of the following probabilities:

$$p_{11}(x, x + t)$$

$$p_{22}(x, x + t)$$

(iii) Write down an integral expression for  $p_{12}(x, x + t)$  in terms of transition rates and the probabilities in (ii). **(10marks)**

2. A friendly society provides the following benefits:

(i) On sickness, \$40 per week for the first 26 weeks and \$50 per week for the next 26 weeks. No sickness benefit is payable after age 65.

(ii) On attaining age 65 or immediately on earlier death, the sum of \$3,000.

Members contribute \$1 per week, ceasing at age 65 or earlier death, but this is waived during periods of sickness (whether benefit is payable or not.) Calculate the reserve which should be held for a member aged 50, using the following basis:

English Life Table No.12 - Males, 4% per annum interest, Manchester Unity Sickness Experience 1893-97 (AHJ). There is no waiting period, and off-periods are as in "Formulae and Tables for Actuarial Examinations". **(10marks)**

#### Question 4 (20marks)

1. A life office issues policies to lives aged under 50 providing the following benefits:

(i) On becoming permanently disabled before age 50, an annuity of \$1,000 per annum payable weekly for life and \$10,000 immediately on death, and immediately on death before age 50 while not permanently disabled, \$15,000. Calculate the office annual premium, payable weekly and ceasing on death, on permanent disability or on reaching age 50, for a life aged 48 if the office uses the following basis:

Mortality: the independent rates of mortality of those not permanently disabled are those of A1967-70 ultimate; the permanently disabled are subject to the mortality of English Life Table No.12 - Males with the age rated up by 6 1/2 years; Permanent disability: a constant independent rate of 0.007; Interest: 4% per annum; Expenses: 2 1/2% of all office premiums, plus \$60 at the issue date. **(10marks)**

2. Write the mean present values of the following assurances in-terms of commutation functions. **(10marks)**

(i) Temporary assurance payable immediately on death

(ii) An endowment assurance of term  $n$  years

(iii) An  $n$  year deferred assurance

- (iv) Annuity payable annually in advance
- (v) Annuity payable annually in arrear

**Question 5 (20 marks)**

1. A policy issued by a life office to a male life aged exactly 35 is subject to level weekly premiums ceasing at exact age 65. If the man has been sick for 6 months or more when a premium falls due, the premium is waived. The policy provides the following benefits:
  - (a) On survival to exact age 65, an annuity of £5,000 per annum payable monthly in advance.
  - (b) On death before age 65, a return of all premiums paid (including those waived during sickness) together with compound interest at 4% per annum to the date of death.There is no waiting period and the off periods are the same as those underlying the tables in Formulae and Tables for Actuarial Examinations. Calculate the weekly premium.  
Basis: English Life Table No.12-Males, Manchester Unity Sickness Experience 1893-97, Occupation Group AHJ, interest 4% per annum, no expenses. **(8marks)**
2. (a) Ten years ago, a man then aged exactly 30 effected an insurance policy providing sickness benefits of £100 per week for the first six months of sickness, £50 per week for the remainder of the first year and £30 per week thereafter, with benefit ceasing at age 60. Calculate the weekly premium payable to age 50 on the following basis:  
Mortality: English Life Table No. 12 - Males;  
Sickness: Manchester Unity 1893-97, Occupation Group AHJ;  
Interest: 4% per annum;  
Expenses: 10% of each premium.
  - (b) The man now wishes to alter his policy so that premiums will in future be waived during all periods of sickness. Calculate the revised premium payable assuming that the alteration basis follows the premium basis above.Note. Expenses are incurred even when premiums are waived **(12marks)**