

KISII UNIVERSITY  
BSc. ACTUARIAL SCIENCE  
LIFE AND HEALTH INSURANCE

BACS 320  
EXAM

January, 2022

ATTEMPT ALL QUESTIONS IN SECTION ONE AND ANY TWO  
OTHER QUESTIONS IN SECTION TWO.

**TIME: 2 HOURS**

**SECTION ONE (30 marks)**

## 1 Question One

a) Given  $q_{70} = 0.010413$  and  $q_{71} = 0.011670$ , calculate  ${}_{0.7}q_{70.6}$  under the assumption of

- i) constant force of mortality
- ii) Uniform distribution of death (6 marks)

b) Show that

$$A_x = vq_x + vp_x A_{x+1}$$

(3 marks)

c) Calculate the exact value of  $\bar{A}_{70:\overline{1}|}$  assuming the *CFM* between consecutive integer ages

Basis: Mortality *ELT* 15 (males)

Interest : 7.5%

(5 marks)

d) An actuary assumes that the future lifetime for a given class of risk,  $T|\mu \sim \exp(\mu)$  and the variability of  $\mu$  between the different classes of risk can be modelled using the gamma distribution defined below.

$$g(\mu) = \frac{1}{\Gamma(\alpha)\beta^\alpha} e^{-\frac{\mu}{\beta}} \mu^{\alpha-1}$$

Find

- i) The mean and variance for the mixed model.
- ii) Criticize the exponential model and explain the importance of exponential mixtures in life insurance.

(8 marks)

e) An annuity is payable continuously throughout the lifetime of a person now aged exactly 50, but for at most 20 years. The rate of payment at all times  $t$  during the first 10 years is £50,000 *pa*, and thereafter it is £7,000 *pa*.

The force of mortality of this life is 0.02 *pa* between the ages of 50 and 60, and 0.03 *pa* between the ages of 60 and 70. Calculate the expected present value of this annuity assuming a force of interest of 0.04 *pa*. (5 marks)

## SECTION TWO (40 marks)

### 2 Question Two

a) A life office sells 5 – year term assurance policies to lives aged 60. Each policy has a sum assured of *Kshs.*180,000 payable at the end of year of death. Premiums of *Kshs.*40,000 are payable annually in advance throughout the 5 – year term or until earlier death.

Let  $L$  denote the present value of the insurer's loss on one of these policies, at policy outset, ignoring expenses.

- i) Write down an expression for  $L$ .
- ii) Assuming *AM92* Ultimate mortality and 7½% *pa* interest, calculate the expected value and standard deviation of  $L$ .

(12 marks)

- b) Assuming that the force of mortality between consecutive integer ages is constant in the *AM92 Ultimate* tables, calculate the exact value of  $\bar{A}_{40:\overline{3}|}$  using a rate of interest of 4% *pa.* (8 marks)

### 3 Question Three

- a) A special term assurance policy is to be issued to a life currently aged 52 exact. The policy term is 8 years, and the sum assured is paid at the end of the year of death. The benefit is 120,000 in the first year, increasing by 10,000 at the end of each year, so that if death occurs in the final policy year 190,000 will be paid.

Calculate the *EPV* of this policy benefit using the following basis:

Mortality: *AM92 Ultimate*

Interest: 4% *pa* (7 marks)

- b) Calculate the variance of the present value of benefits under an annuity payable to a life aged 35 exact. The annuity has payments of 1 per annum payable continuously for life.

Basis:

Mortality  $\mu = 0.02$  throughout

Interest  $\delta = 0.05$  (7 marks)

- c) Assuming *AM92* mortality and  $5\frac{1}{2}\%$  interest
- i) calculate the value of  $A_{38:\overline{3}|}$
- ii) estimate using linear interpolation the constant force of mortality which will give the value obtained in (i) (6 marks)

### 4 Question Four

- a) An actuary models lifetime distribution of each risk in a particular portfolio using an exponential distribution with parameter  $\mu$ . She believes the variability of the parameter  $\mu$  among these risks can be modelled using an inverse gaussian distribution of parameter  $(\delta, \gamma)$ .

i) Construct the aggregate (mixed) model for the portfolio

- ii) using the conditional expectation approach, find explicit expressions for the mean and variance of the mixed model.

iii) Give the expressions for the method of moment estimators for the parameters.

(20 marks)

## 5 Question Five

a) A man aged exactly 42 purchases a whole life assurance policy with a sum assured of 5,000 payable immediately on death.

i) Write down an expression for the random variable representing the present value of the benefits from this policy. (1 mark)

ii) Show that the variance of this random variable is  $5000^2 \left( {}^2\bar{A}_{42} - (\bar{A}_{42})^2 \right)$ .  
(2 marks)

iii) Calculate the variance of the present value of the benefits from this assurance policy, assuming AM92 Ultimate mortality and 4% pa interest.  
(2 marks)

b) Let  $K$  denote the curtate future lifetime random variable of a life aged exactly  $x$ .

i) Describe the benefit whose present value random variable is:

$$W = \left\{ \begin{array}{ll} 10,000\ddot{a}_{\overline{k+1}|} & \text{if } K < 10 \\ 10,000\ddot{a}_{\overline{10}|} & \text{if } K \geq 10 \end{array} \right\}$$

(1 mark)

ii) Show that

$$A_{x:\overline{n}|} = 1 - d\ddot{a}_{x:\overline{n}|}$$

(2 marks)

iii) Calculate the expected present value and the standard deviation of the present value of the benefit in (i), assuming:

**a)** a force of interest of 0.04 pa

**b)** the life is subject to a constant force of mortality of 0.02 pa. (8 *marks*)

c) Given  $q_{70} = 0.010142$ ,  $q_{71} = 0.015462$ ,  $q_{72} = 0.021743$ ,  $q_{73} = 0.027323$   
find

**i)**  ${}_3p_{70}$

**ii)**  ${}_4q_{70}$  (3 *marks*)