KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

SPECIAL EXAMINATION

THIRD YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF ACTURIAL SCIENCE

SECOND SEMESTER 2021/2022

(JULY, 2022)

BACS 321: LIFE CONTIGENCIES I

STREAM: Y3 S2 **TIME: 2 HOURS**

DAY: MONDAY, 8.00 AM – 10.00 AM **DATE: 25/07/2022**

INSTRUCTIONS:

1. Do not write anything on this question paper.
2. Answer ALL Questions in section A (Compulsory) and any other TWO

SECTION A (30 MARKS)

QUESTION ONE (30 MARKS)

(a) Derive a binomial distribution given $(a+b)^n$,then let a=1 and b=ɵ (3marks)

(b) Use both cumulative and change of variable technique to determine the pdf of y given $y = -\frac{1}{\lambda}\boldsymbol{lnx} where\ \lambda > 0$ and x is uniform distribution with [0,1] (7marks)

(c) Express kummers hypergeometric function in terms of gamma, beta and integral form simplifying as much as possible (10marks)

(d) Suppose $\hat{p}$ is normal distribution with mean and variance ($p, \frac{p(1-p)}{n}$).find variance f($\hat{p}$) given that $f(p) = \frac{p}{1-p}$ (3marks)

(e) perform chi-square text on the following data to test whether there is relationship between blood group and conducting the diseases

Disease /Blood group	AB	B	O
Cancer	50	50	50
Heart attack	100	100	50
Respiratory disease	50	50	50

Let confidence interval be of 5% (7marks)

SECTION B

QUESTION TWO (20 MARKS)

(a) The data bellow is obtained from two groups of data which were graduated and the following results were obtained:

Data from group X

Age x	$\hat{q}$	$\mathring{q}$
30	0.000557	0.000460
31	0.000645	0.000508
32	0.000497	0.000548
33	0.000474	0.000578
34	0.000372	0.000600
35	0.000757	0.000616

Data from group Y

Age x	$\hat{q}$	$\mathring{q}$
30	0.00557	0.000387
31	0.00645	0.000428
32	0.00497	0.000473
33	0.00474	0.000523
34	0.00372	0.000579
35	0.00757	0.000640

Compare the smoothness and fit to data of the following graduated data between X and

(5marks)

(b) Describe the differences between deterministic and stochastic models. (5marks)

(c) State the benefits of modeling in actuarial work. (5marks)

(d) Given the following equation

$\mu_{x+t} = \frac{1}{85-t} + \frac{3}{105-t}$ $0 \leq t \leq 85$ Calculate $_{20}\mathbf{p_x}$ (5marks)

QUESTION THREE (20 MARKS)

(a) A population with limiting age 100 has the following survival function :

${}_tp_0 = (1 - \frac{t}{100})^{0.5} \{for\ x \geq 0$

Calculate the complete expectation of a life at birth (5marks)

(b) List key steps involved in developing an actuarial model (5marks)

(c) Comment on considerations which would apply if you were developing a model of the spread of a newly discovered disease (2marks)

(d) Use change of variable technique to find pfd of $\boldsymbol{y}$given $y = ln\frac{x}{z}$ where z is a constant and had a pdf $f(x) = \frac{\lambda}{z}(\frac{z}{x})^{\lambda+1}$,x>z (5marks)

(e) Let $X = \frac{B}{Y}$ and x is gamma distribution with one parameter ,find pdf of **y** (3marks)

QUESTION FOUR (20 MARKS)

(a) Discuss the advantages and disadvantages of actuarial models (7marks)

(b) Consider $\boldsymbol{y} = \boldsymbol{x}^{\frac{1}{k}}$ where **k** is a constant and **x** is exponential distribution with parameter **λ.**Find the pdf of **y.** (5marks)

(c) A researcher is reviewing a study published in a medical journal into survival after a major operation. The journal only gives the following summary information

- the study followed 16 patients from the point of surgery
- the patients were studied until the earliest of five years after the operation, the end of the study or the withdrawal of the patient from the study

The Nelson-Aalen estimate, *S*(*t*), of the survival function was as follows:

Duration since operation t(years)	S(t)
$0 \leq t < 1$	1
$1 \leq t < 3$	0.9355
$3 \leq t < 4$	0.7122
$4 \leq t < 5$	0.6285

Calculate the number of deaths which occurred, classified by duration since the operation. (8marks)

QUESTION FIVE (20 MARKS)

(a) Given the following pareto distribution $f(x) = \frac{z\lambda^z}{(x+\lambda)^{z+1}}$,transform it to find pdf of y if $y = x^B$ for B>0 (5marks)

(b) Name and describe graphically distributions used in fully parametric models (5marks)

(c) The decrement table extract below is based on the historical experience of a very large multinational company's workforce.

Age x	Number of employees $(al)_x$	Deaths $(ad)_x^d$	Withdrawal $(ad)_x^w$
40	10,000	25	120
41	9,855	27	144
42	9,684		

Recent changes in working conditions have resulted in an estimate that the annual independent rate of withdrawal is now 75% of that previously used. Calculate a revised table assuming no changes to the independent death rates, stating your results to one decimal place. (5marks)

(d) A toy manufacturer is testing the lifetime of its new electric children's toy. 500 are set going at 9 a.m. one morning on test rigs plugged into the electricity supply and are run until 5 p.m. the next day or until they fail, whichever comes first. Unfortunately the cleaner unplugged a test rig on which 17 toys were still working at 7 p.m. on the first evening in order to plug his floor polisher in. Then, as he left work three hours later, he took three of the still working toys for his children to play with. Of the other480 toys it was found that 12 failed after four hours, 25 failed after 11 hours and a further 8 failed after 31 hours.

Calculate the Nelson-Aalen estimate of the survival function. (5marks)