KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

SPECIAL EXAMINATION

THIRD YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS /BACHELOR OF EDUCATION SCIENCE AND ARTS

FIRST SEMESTER 2021/2022

(JULY, 2022)

MATH 312: REAL ANALYSIS 1

STREAM: Y3 S1 **TIME: 2 HOURS**

DAY: WEDNESDAY, 3.00 PM – 5.00 PM **DATE: 20/07/2022**

INSTRUCTIONS:

1. ***Do not write anything on this question paper.***
2. ***Answer Question ONE (Compulsory) and any other TWO Questions***

QUESTION ONE (30 MARKS)

(a) Define a ‘’bounded set’’ of real numbers (2marks)

(b) (i) What is a null sequence (1mark)

(ii) Suppose $y_n \to l$ as $n \to \infty$ and $z_n \to l$ as $n \to \infty$. If $y_n \leq x_n \leq z_n$,

prove that $x_n \to l$ as $n \to \infty$. (4marks)

(c) Prove that for any number $x \in \mathbb{R}$, $-|x| \leq x \leq |x|$ (3marks)

(d) Prove that $\sqrt{3}$ is irrational. (5marks)

(e) Prove that between any two rational numbers there is another rational number. (4marks)

(f) Use mathematical induction to prove that $10^{2n-1} + 1$ is divisible by 11

for all $n \in \mathbb{N}$ (5marks)

(g)(i) If $a, b, c \in \mathbb{R}$, Prove the following property of an ordered field:

$a + b = c + b$ implies $a = c$. (2marks)

(ii) Show that $-(-y) = y$, $\forall\, y \in \mathbb{R}$ (4marks)

QUESTION TWO

(a) (i) Define the term 'Supremum' for a set of real numbers. (1mark)

(ii) Show that the Supremum of a set of real numbers (if it exists) is unique. (5marks)

(iii) Distinguish between 'Infimum' and 'minimum' values of a set of real numbers.

(2marks)

(b) State the completeness axiom of the set of real numbers. (2marks)

(c) Prove that the set of natural numbers is unbounded above. (5marks)

(d) Show that if a and b are positive real numbers , then there exist a positive integer n such that $na > b$. (5marks)

QUESTION THREE

(a) (i) Let (X, d) be a metric space and $A \subseteq X$. Define a limit point of A. (2marks)

(ii) If (X, d) is a metric space and $k > 0$ is a real number, show that (X, d_k)

is a metric space where $d_k(x, y) = kd(x, y)$ (10marks)

(b) Define an interior point of a set A if (X, d) is a metric space. (2marks)

(c) (i) What is meant by an open set given that (X, d) is a metric space? (2marks)

(iii) Prove that E^0 is always an open set (4marks)

QUESTION FOUR

(a) (i) Evaluate $\lim\limits_{n\to\infty}\left(\frac{5n^3-4n}{7n^3-2n^2+6}\right)$ (2marks)

(ii) Show that $\lim\limits_{n\to\infty}\left(\frac{4n+1}{n+2}\right) = 4$ (3marks)

(b) (i) Define a Cauchy sequence (2marks)

(ii) show that any Cauchy sequences is always bounded. (5marks)

(c) State and prove the Bolzano-Weirstrass theorem (4marks)

(d) Prove that the limit of a sequence is unique. (4marks)

QUESTION FIVE

(a) (i)When is a function f continuous at a point c ? (2marks)

(ii) Show that the function $f(x) = \begin{cases} x+2, & x \geq -3 \\ 3x+8, & x < -3 \end{cases}$ is continuous at $x = -3$ (3marks)

(b) State and prove Rolle's theorem. (5marks)

(c) (i) State and prove the intermediate value theorem for continuous functions. (4marks)

(ii) Use the intermediate value theorem to show that the function

$f(x) = x^2 + x - 3$ has a root in the interval $(1,2)$ (3marks)

(d) Prove that $\lim_{n \to 1} 2x + 3 = 5$ using $\varepsilon - \delta$ definition. (3marks)