KISII UNIVERSITY

THIRD YEAR EXAMINATION FOR BACHELOR OF EDUCATION SCIENCE AND BSC PURE MATHEMATICS

MATH 313: REAL ANALYSIS II **STREAM: Y3S2**

TIME: 2 HOURS

INSTRUCTIONS

1.Do not write anything on this question paper.

2.Answer questions ONE (compulsory) and and any other TWO questions.

QUESTION ONE (30 MARKS)

(a) Let $\{x_n\}$ be a sequence of elements in $\mathbb{K}$. Define convergence of $\{x_n\}$. [2 mks]

(b) Let $\{a_n\}$ and $\{b_n\}$ be convergent sequences of elements in $\mathbb{K}$ with limits a and b respectively. Show that $\lim_{n\to\infty}\{a_n + b_n\} = (a + b)$ [7 mks]

(c) Use ratio test to find the interval and radius of convergence of $\sum_{n=0}^{\infty}\frac{(-3)^n}{n+1}(x-2)^2$. [7 mks]

(d) State without proof the Lagrange's mean value theorem [2 mks]

(e) Assuming the sequence $\sum_{n=1}^{\infty} x_n$ converges, show that $\lim_{n\to\infty} x_n = 0$. [6mks]

(f) Find the values of X for which the series $\sum_{n=1}^{\infty}\frac{(2x+1)^n}{n^2}$ is convergent. [6 mks]

QUESTION TWO (20 MARKS)

(a) Using root test method show that the following series converge:

(i) $\sum_{n=1}^{\infty}\frac{n^2}{2^n}$ [6 mks]

(ii) $\sum_{n=1}^{\infty}(-1)^n(x-1)^n$ [6 mks]

(b) Show that if $\sum_{n\in\mathbb{N}} \mathcal{Z}_n$ is absolutely convergent, then $\sum_{n\in\mathbb{N}} \mathcal{Z}_n$ is convergent. [8 mks]

QUESTION THREE (20 MARKS)

(a) Show that the function f defined on $[0,1]$ by $f(x) = \begin{cases} 1, x \text{ is rational} \\ 0, x \text{ is irrational} \end{cases}$

is not integrable on [0,1]. [7 mks]

(b) Let f be a real-valued function which is bounded on $[a, b]$, prove that

$\underline{\int_a^b} f(x)dx \leq \overline{\int_a^b} f(x)dx.$ [6 mks]

(c) Show that if $f(x) = k$ is a constant function on $[a, b]$, then f is Riemann integrable (i.e $f \in \Re[a, b]$) and find its integral. [7 mks]

QUESTION FOUR (20 MARKS)

(a) Using ratio test determine convergence of the infinite series $\sum_{n=1}^{\infty} \frac{nx^n}{2^n}$ and hence or otherwise find the radius and interval of convergence. [7 mks]

(b) Find the values of x for which the series $\sum_{n=1}^{\infty} n^n x^n$ is convergent using root test method and find the interval of convergence. [7 mks]

(c) Use comparison test to show that $\sum_{n=1}^{\infty} \frac{1}{n.2^n}$ is convergent. [6 mks]

QUESTION FIVE (20 MARKS)

(a) Prove that a bounded monotonic function is a function of bounded variation. [6 mks]

(b) Define without proof the Cauchy's criterion for Riemann-Stieltjes Integrability. [2 mks]

(c) Let f be a continuous function on $[a, b]$, show that f is Riemann- Stieltjes integrable on $[a, b]$. [6 mks]

(d) Prove that $f(x) = \sin x$ is Riemann Integrable over $\left[0, \frac{\pi}{2}\right]$. [6 mks]