KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

SPECIAL EXAMINATION

THIRD YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE MATHEMATICS/ BACHELOR OF SCIENCE APPLIED STATISTICS

FIRST SEMESTER 2021/2022

(JULY, 2022)

MATH 318: CALCULUS III

STREAM: Y3 S1 **TIME: 2 HOURS**

DAY: THURSDAY, 11.30 AM – 1.30 PM **DATE: 21/07/2022**

INSTRUCTIONS:

1. Do not write anything on this question paper.

2. Answer Question ONE (Compulsory) and any other TWO Questions.

QUESTION ONE (30MARKS)

(a) (i) State Lagrange's mean value theorem. (2marks)

(ii) Verify Lagrange's mean value theorem for the function $f(x) = x^2 - x - 12 \ in$

$[0 \leq x \leq 1]$ (4marks)

(b) If $u(x, y) = 2xsiny + 3y^2cosx$, find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}$. (3marks)

(c) (i) Define an infinite series. (1 mark)

(ii) Test the convergence of the following series:

(I) $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + - - - - \infty$ (3marks)

(II) $1 + 5 + 9 + 13 + 17 + - - - - - - - - - \infty$ (3marks)

(d) Show that $\int_0^1 \frac{dx}{\sqrt{1-x}}$ is an improper integral and evaluate it. (5marks)

(e) Test for an extremum the functional (4marks)

$$I[y(x)] = \int_0^1 (xy + y^2 - 2y^2y')dx \ , \qquad y(0) = 1 \ , \ y(1) = 2$$

(f) Show that $f(x) = x^3 + \sin x$ is an odd function. (3marks)

(g) Prove that $\Gamma(1) = 1$. (2marks)

QUESTION TWO

(a) Prove that for every convergent series $\sum u_n$, $\lim_{n\to\infty} u_n = 0$. (4marks)

(b) Test for the convergence of the following series using Cauchy's fundamental test. (3marks)

$$\sum_{n=1}^{\infty} \frac{n}{n+1}$$

(c) (i) Define a positive term series. (1mk)

(ii) Prove that if $\sum u_n$ is a positive term series such that $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = k$, then the series is convergent if $k < 1$. (5marks)

(d) Test for convergence of the following series using D'Alembert's ratio test:

(i) $\sum \frac{2^n}{n^3}$ (3marks)

(ii) $\sum \frac{n!}{n^n}$ (4marks)

QUESTION THREE

(a) Evaluate the following improper integrals without using tables and state whether they converge or diverge:

(i) $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ (7marks)

(ii) $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$ (6marks)

(b) Prove that the shortest distance between two points is along a straight line. (7marks)

QUESTION FOUR

(a) (i) Let $f = f(x, y)$ be a function of two independent variables. State two conditions required for the function f to have extreme values. (2marks)

(ii) Explain how you will distinguish between the maximum and minimum point of the function $f(x, y)$. (2marks)

(b) Find the stationary points of $f(x, y) = x^2 + y^2$ subject to the constraint $3x + 2y = 6$. (8marks)

(c) (i) State the Euler's equation. (2marks)

(ii) Solve the Euler's equation for $\int_{x_0}^{x_1}(x + y')y'dx$ (6marks)

QUESTION FIVE

(a) Find C of the Lagrange's mean value theorem for the function $f(x) = e^x$ in the interval $[0 \leq x \leq 1]$. (4marks)

(b) Express $\frac{dw}{dt}$ using chain rule as a function of t given that

$w = x^2 + y^2$, $x = \cos t$, $y = \sin t$. (4marks)

(c) Define the following:

(i) an even function. (1 mark)

(ii) an odd function. (1 mark)

(d) Expand $f(x) = x^2$, $-\pi < x < \pi$ in a Fourier series. (10 marks)