



UNIVERSITY EXAMINATIONS
SPECIAL EXAMINATION
FOURTH YEAR EXAMINATION FOR THE AWARD OF DEGREE OF
BACHELOR OF SCIENCE MATHEMATICS/BACHELOR OF EDUCATION
SCIENCE /ARTS
FIRST SEMESTER 2021/2022
(JULY, 2022)

MATH 420: PARTIAL DIFFERENTIAL EQUATIONS I

STREAM: Y4 S1

TIME: 2 HOURS

DAY: THURSDAY, 3.00 PM – 5.00 PM

DATE: 28/07/2022

INSTRUCTIONS:

- 1. Do not write anything on this question paper.**
- 2. Answer Question ONE (Compulsory) and any other TWO Questions.**
- 3 Show all the relevant working.**

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Form a partial differential equation from
 - i) $ax^2 + by^2 + z^2 = 1$ (4marks)
 - ii) $z = f(x + y + z, x^2 + y^2 + z^2)$ (4marks)
- b) Verify that the equation $(x^2z - y^3)dx + 3xy^2dy + x^3dz = 0$ is integrable. (4marks)
- c) Solve the partial differential equation
$$p^2 + q^2 = x + y$$
(4marks)
- d) Solve: $\frac{\partial^3 z}{\partial x^2 \partial y} = \cos(2x + 3y)$ (4marks)
- e) Using multipliers solve:

$$(mz - ny) \frac{\partial z}{\partial x} + (nx - lz) \frac{\partial z}{\partial y} = ly - mx. \quad (5\text{marks})$$

- f) Find the orthogonal trajectories on the family of curves $y + 1 = ax$ where a is a constant (5marks)

QUESTION TWO (20 MARKS)

- a) Derive the solution for the Lagrange equation of the form $Pp + Qq = R$ (6marks)
- b) Find the general solution of the equation:
 $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$ (5marks)
- c) Solve the partial differential equation $yq - xp = z$ (3marks)
- d.) Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$ (6marks)

QUESTION THREE (20 MARKS)

- a) Solve the following non-linear PDEs
- i) $\sqrt{p} - \sqrt{q} = 1$ (3marks)
- ii) $p(1 + q) = qz$ (3marks)
- b) Solve the differential equation: $x^2p^2 + y^2q^2 = z^2$ (6marks)
- c) Form the PDE from $x^2 + y^2 = (z - c)^2 \tan^2 \alpha$ (5marks)
- d) Form the PDE from: $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ (3marks)

QUESTION FOUR (20 MARKS)

- a) Show that the direction cosines of the tangent at the point (x, y, z) to the conic $ax^2 + by^2 + cz^2 = 1, x + y + z = 1$ are proportional to $(by - cz, cz - ax, ax - by)$. (5marks)
- b) Find the integral curves of the equations
 $\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}$ (5marks)
- c) Using Char pit's method solve: $2(z + xp + yq) = yp^2$ (7marks)
- d.) Solve $\frac{\partial^2 z}{\partial x \partial y} = x^2y$ subject to the condition $z(x, 0) = x^2$ and $z(1, y) = \cos y$. (5marks)

QUESTION FIVE (20MARKS)

- a) Find the surface which intersects with the surfaces of the system $z(x + y) = c(3z + 1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1, z = 1$. (8marks)
- b) Using the method of separation of variables, solve;
 $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$ where $u(x, 0) = 6e^{-3x}$ (7marks)
- c) Form the partial differential equation of:
 $f(x^2 + y^2 + z^2, z^2 - 2xy) = 0$ (5marks)