KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

SPECIAL EXAMINATION

FOURTH YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

SECOND SEMESTER 2021/2022

(JULY, 2022)

MATH 423: PARTIAL DIFFERENTIAL EQUATIONS II

STREAM: Y4 S2 **TIME: 2 HOURS**

DAY: WEDNESDAY, 8.00 AM – 10.00 AM **DATE: 20/07/2022**

INSTRUCTIONS:

1. Do not write anything on this question paper.
2. Answer Question ONE (Compulsory) and any other TWO Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Classify the following pdes as either hyperbolic, parabolic or elliptic:

i) $2\frac{\partial^2 u}{\partial t^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 3\frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} + y\frac{\partial u}{\partial x}$ (3marks)

ii) $\frac{\partial^2 y}{\partial t^2} = a^2\frac{\partial^2 y}{\partial x^2}$ (3marks)

iii) $u_{xx} + 5u_{xy} + 2u_{yy} = \sin(3x + 2y)$ (3marks)

iv) $xu_{xx} + xu_{xy} + yu_{yy} = 0$ (3marks)

b) Solve the following equations:

i) $(25D^2 - 40DD' + 16D'^2)z = 0$ (3marks)

ii) $r + t + 2s = 0$ (3marks)

iii) $r = a^2 t$ (3marks)

iv) $r - 7s + 12t = e^{x+2y}$ (4marks)

c) Solve: $y' = 0.75y$ with $y(0) = 16$ given $y' = \frac{dy}{dt}$ (3marks)

d) Solve: $\frac{du}{dx} = 5x$ given that $x = 0\ when\ u = 10$ (2marks)

QUESTION TWO (20MARKS)

a) Solve the wave equation: $\frac{\partial^2 y}{\partial t^2} = 16\frac{\partial^2 y}{\partial x^2}$ using separation of variables, given that $0 \leq x \leq 2$; $t > 0$. State whether it is hyperbolic, elliptic or parabolic subject to the boundary conditions: $y(0,t) = 0$; $y(2,t) = 0$. (10marks)

b) Solve the following pdes using separation of variables:

i)$\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$given the boundary conditions $u(0,y) = 8e^{-3y}$where $u = u(x,y)$. (5marks)

ii)$\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$given that $u(0,y) = 8e^{-3y} + 4e^{-5y}$ (5marks)

QUESTION THREE (20MARKS)

a) A bar whose surface is insulated has a length of 3 units and diffusivity is 2units. If its ends are kept at temperature 0 units and initial temperature $u(x,0) = 5sin4\pi x - 3sin8\pi x + 2sin10\pi x$.

i)Find the temperature at position x at time t, i.e. $u = u(x,t)$ (13marks)

ii)Evaluate$u\ when\ x = \frac{5}{4}; t = 10\ sec$ (2marks)

b) Solve: $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = 2sin(3x + 2y)$ (5marks)

QUESTION FOUR (20MARKS)

a) Solve:

i)$(D^2 + 3DD' + 2D'^2)z = x + y$ (6marks)

ii)$(D^3 - 2D^2D')z = 2e^{2x} + 3x^2y$ (6marks)

iii)$(D^2 - DD' - 2D'^2)z = (y-1)e^x$ (6marks)

b) Solve: $(D^3 - 4D^2D' + 3DD'^2)z = 0$ (2marks)