KISII UNIVERSITY

FIRST YEAR SEMESTER ONE EXAMINATION FOR THE

DEGREE OF MASTER OF SCIENCE- PURE MATHEMATICS

MATH 801- FUNCTIONAL ANALYSIS I

TIME- 3 HOURS

STREAM: MSC PURE MATHEMATICS

INSTRUCTIONS: Answer question one (compulsory) and any other two

questions

QUESTION ONE (30 MARKS)- COMPULSORY

a) i) Define a normed linear space (3mks)

ii) Let X be a normed linear space.Define a function $d : X \times X \longrightarrow \mathbb{R}$ by $d(x, y) = \|x - y\| \, \forall \, x, y \in X$. Show that d is a metric on X . Further more show that d is invariant i.e $d(x, y) = \rho(x + z, y + z)$ (10mks)

b) i) Define the term strong convergence in a normed linear space (3mks)

ii) Let l_n^p be the space of all $n-$tuples of complex or real numbers. Show show that $\|\|_p$ is a norm on l_n^p (10mks)

c) Define the following terms

i) Linear hull (2mks)

ii) Manifold (2mks)

QUESTION TWO (20 MARKS)

a) Define a bounded linear transformation (2mks)

b) Let $X, \ Y$ be normed linear spaces and $T : X \longrightarrow Y$ be a linear transformation.

Then the following statements are equivalent

i) T is bounded (4mks)

ii) T is uniformly continuous (3mks)

iii) T is continuous at some point $x_0 \in X$. (4mks)

c) Let X be normed linear space over a field K, M be a linear subspace of X and f be a bounded linear functional on M. Prove that there exists a bounded linear functional F on X such that F$|_M = f$. Further $\|F\| = f$ (7mks)

QUESTION THREE (20 MARKS)

a) State the principle of uniform boundedness (2mks)

b) i)Define the term weak convergence in normed linear spaces (2mk)

ii) Prove that if $\{x_n\}$converges weakly, then the weak limit is unique (6mks)

ii) Define the term a Schaunder basis (2mks)

c) i) Define the term a subspace (1mk)

ii) Let M be a linear subspace of a normed linear space X and $\overline{M}$ denote the closure of M with respect to the norm determined metric on X.Show that $\overline{M}$ is a linear subspace of X (7mks)

QUESTION FOUR (20 MARKS)

a) Let X be a linear space and M be a subspace of X. Prove that for any $x \in X$,

$[x] = x + M$ (9mks)

b) i) What is a linear hull? (2mks)

ii) Let X be a normed linear space and $A \subseteq X$, Prove that the closed linear hull

of A is the closure of the linear hull of A (9mks)