KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF
THE DEGREE OF MASTER OF SCIENCE PURE MATHEMATICS
FIRST SEMESTER 2021/2022
(JULY-AUGUST, 2022)

MATH 805: GROUP THEORY I

STREAM: Y1S1 **TIME: 3 HOURS**

DAY: THURSDAY, 8.00 AM – 11.00 AM **DATE: 04/08/2022**

INSTRUCTIONS:

1. ***Do not write anything on this question paper.***
2. ***Answer Question one (Compulsory) and Any Other TWO Questions.***

QUESTION ONE (30 MARKS)- COMPULSORY

a) i) Define a simple group. (1 mark)

ii) Name any two families of simple groups (2 marks)

b) i) Define a composition series and a subnormal series of a group G (2 marks)

ii) Prove that every finite group Ghas a composition series (8marks)

iii) Explain why the additive group of integers $\mathbb{Z}$does not have a composition series

(3 marks)

c) State and proof the Jordan – Holder theorem (8 marks)

d) Let $(G, *)$ be a group. Prove that the identity element in $G$is unique (5mks)

QUESTION THREE (20 MARKS)

a) Let G be a group.

i) Define a lower central series of G. (1 mark)

ii) Define an upper central series of G. (1 mark)

b) i) Define a nilpotent group. (1 mark)

ii) Show that every nilpotent group is soluble. (4 marks)

iii) By a counter example show that not all soluble groups are nilpotent (2mks)

c) i) Define a p-group. (2 marks)

ii) Show that every p-group is nilpotent. (5 marks)

d) Show that any finite p-group is nilpotent (4 marks)

QUESTION FOUR (20 MARKS)

a) i) Define a soluble group. (2 marks)

ii) Show that if $n \geq 5$, S_n is not soluble. (4 marks)

b) Let$H \Delta G$. Show that if both H and G/H are soluble, the G is soluble. (4 marks)

c) i) Show that all finite abelian groups are soluble. (4 marks)

ii) Show that a finite group G is soluble if it contains a normal subgroup K such that K and G/K are soluble (6marks)

QUESTION FIVE (20 MARKS)

a) i) Define what is meant by the external direct product of two groups H and K (1 mark)

ii) Show that for any groups H and K, $H \times K \cong K \times H$ (4 marks)

b) Show that $H \times I$ and $I \times K$ are normal subgroups of H×K; these two subgroups generate $H \times K$ and their intersection is (l, l) (4 marks)

c) Given the group (G, $+_4$) where $G=\{0,1,2\}$. Find the generator of G (3 marks)

d) Let Z be the set of integers under the binary operation * defined by $a*b=a+b+1$, for all $a,b \in Z$. Show that Z is a group under* as defined (8 marks)