KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF
THE DEGREE OF MASTER OF SCIENCE APPLIED MATHEMATICS
FIRST SEMESTER 2021/2022
(JULY-AUGUST, 2022)

MATH 837: PARTIAL DIFFERENTIAL EQUATIONS I

STREAM: Y1S1 **TIME: 3 HOURS**

DAY: THURSDAY, 8.00 AM – 11.00 AM **DATE: 04/08/2022**

INSTRUCTIONS:

1. ***Do not write anything on this question paper.***
2. ***Answer Question one (Compulsory) and Any Other THREE Questions.***

QUESTION ONE (15 MARKS)

a) Solve the wave equation using separation of variables method and state whether it is hyperbolic, elliptic or parabolic.
$\frac{\partial^2 u}{\partial t^2} = 16\frac{\partial^2 u}{\partial x^2}$ subject to the conditions $y(0,t) = 0, y(3,t) = 0,\ t > 0$ and $0 \leq x \leq 3$. (7marks)

b) Find the eigenvalues and eigenfunctions for the boundary value problem
$y'' + \lambda\, y = 0, \quad y(0) = 0, \quad y'(\pi) = 0$ (5marks)

c) Compute the Fourier series of:
$f(x) = \begin{cases} 3, & 0 < x < 1 \\ -3, & -1 < x < 0 \end{cases}$ (3marks)

QUESTION TWO (15MARKS)

a) A bar whose surface is insulated has a length of 5 units and diffusivity 2 units. If its ends are kept at temperatures zero units at one end and 10 units at the other end. Given that the initial temperature is $u(x,0) =$

$5sin4\pi x - 3sin10\pi x + 2sin12\pi x$, find the temperature distribution at position x at time t. (9marks)

b) Find the Laplace transform of
$7e^{2t} + 5sint + 6t^3 + 5cos3t + 20$ (6marks)

QUESTION THREE (15MARKS)

a) A flexible string of length 4 units is fixed on the x −axis at $x = 0$ and $x = 4$. The tension of the string is given by an initial shape $y(x,0) = 8sin\pi x - 3sin4\pi x + 7sin6\pi x$. The string is gently released from rest. Find the displacement wave which depends on x and t. (9marks)

b) Solve the following partial differential equation using separation of variables method:
$\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$ given the boundary conditions $u(0,y) = 8e^{-3y}$. (6marks)

QUESTION FOUR (15 MARKS)

a) Find the Laplace inverse transform of
$\frac{s^2-3s+4}{s^3}$ (3marks)

b) Use the Laplace transform to solve:
$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3x = e^{-t}$ (7marks)

c) Find the Fourier transform of $e^{-|x|}$
Hence, show that $\int_0^\infty \frac{sinmx}{1+x^2}dx = \frac{\pi e^{-m}}{2}$, $m > 0$ (5marks)

QUESTION FIVE (15MARKS)

a) Obtain the Green function for the boundary value problem
$y'' = 0, \quad y(0) = 0, \quad y(l) = 0$(6marks)

b) Given the periodic function
$$f(x) = x^2 + x, \qquad -\pi < x < \pi$$
i) Sketch the graph for $-3\pi < x < 3\pi$
ii) Determine the Fourier series
iii) By letting $x = \pi$, show that $\frac{\pi^2}{6} = \sum_{n=1}^{\infty}\frac{1}{n^2}$(9marks)

QUESTION SIX (15MARKS)

a) Distinguish between a variable and a constant coefficient partial differential equation giving an example of each. (2marks)

b) Find the Laplace inverse transform of $sin2tsin3t$ (3marks)

c) Use the Green's function to solve the boundary value problem
$y'' + y = f(x)$, with $y(0) = 0$, $y\left(\frac{\pi}{2}\right) = 0$ (8marks)

d) Classify the following partial differential equation

$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ where $a \neq 0$ (2marks)