KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF
THE DEGREE OF MASTER OF SCIENCE APPLIED MATHEMATICS
FIRST SEMESTER 2021/2022
(JULY-AUGUST, 2022)

MATH 852: RIEMANNIAN GEOMETRY I

STREAM: Y1S1 **TIME: 3 HOURS**

DAY: THURSDAY, 2.00 PM – 5.00 PM **DATE: 04/08/2022**

INSTRUCTIONS:

1. ***Do not write anything on this question paper.***
2. ***Answer Question ONE (Compulsory) and Any Other THREE Questions.***

QUESTION ONE *compulsory* (15MARKS)

a. Show that $A_{pq} - A_{qp} = \frac{\partial A_p}{\partial x^q} - \frac{\partial A_q}{\partial x^p}$ [4marks]

b. Determine the metric tensor in cylindrical coordinates [3marks]

c. Calculate the intrinsic derivatives of

i. An invariant ϕ [2marks]

ii. A^j [3marks]

iii. A^j_k [3marks]

QUESTION TWO (15MARKS)

a. If A^{pq}_r and B^{pq}_r are tensors, prove that their sum and difference are also tensors [4marks]

b. Let A^{pq}_{rst} be a tensor, set $p = t$ and show that A^{pq}_{rsp} is also a tensor [3marks]

c. Prove that the construction of the tensor A^{pq}_q is also a scalar or invariant [4marks]

d. Show that the construction of the outer product of the tensors A^p and B_q is an invariant [4marks]

QUESTION THREE (15MARKS)

a. Differentiate between the covariant and contravariant components of $\vec{A}$ (2 marks)
b. Write the law of transformation for the tensors A^{i}_{jk} and B^{mn}_{ijk} [4marks]
c. Proof that the cylindrical coordinate system is orthogonal [5marks]
d. Represent the vector $\vec{A}= z\hat{i} - 2x\hat{j} + y\hat{k}$ in cylindrical coordinate and thus determine its components [4marks]

QUESTION FOUR (15MARKS)

a. Let $\vec{A}$ be a vector defined with respect to two general curvilinear coordinate system $(u_1u_2u_3)\left(\overline{u_1}\,\overline{u_2}\,\overline{u_3}\right)$, establish a relation between the contravariant components in the first and second system [7marks]
b. Given $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$, express the velocity $\bar{v}$ and acceleration $\bar{a}$ of a particle in cylindrical coordinate system [4marks]
c. Prove that $\frac{d}{dt}\left(e_\rho\right) = +\dot{\phi}e_\phi$ and $\frac{d}{dt}\left(e_\phi\right) = -\dot{\phi}e_\rho$ [4marks]

QUESTION FIVE (15MARKS)

a. A contravariant vector has components a,b,c in rectangular coordinate system. Find the contravariant components in spherical coordinates [7 marks]
b. Show that $\frac{\partial A_p}{\partial x^q}$ is not a tensor even though A_p is a covariant tensor of rank one. [4marks]
c. If $\phi = a_{jk}A^jA^k$, show that we can write $\phi = b_{jk}A^jA^k$ where b_{jk} is symmetric. [4marks]

QUESTION SIX (15MARKS)

a. Show that the square of the elements of arc length in general curvilinear system can be expressed as $ds^2 = \sum_{p=1}^{3}\sum_{q=1}^{3} g_{pq}\,du_p\,du_q$ [5marks]
b. A covariant tensor has components $(xy, 2y - z^2, xz)$ in rectangular coordinates. Find its contravariant components in spherical coordinates [10marks]