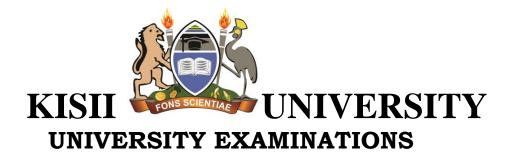
MATH 809



FIRST YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF MASTERS IN PURE MATHEMATICS SECOND SEMESTER 2021/2022 (OCTOBER-JANUARY, 2022)

MATH 809: ABSTRACT INTEGRATION I

STREAM: : Y2S1

TIME: 3 HOURS

DAY: MONDAY, 8.00 AM - 11.00 AM

DATE: 01/03/2022

INSTRUCTIONS:

- 1. Do not write anything on this question paper.
- 2. Answer Question One (Compulsory) and Any Other Two Questions.

SECTION A [30 MARKS]

QUESTION ONE

- a. Show that $P.V \int_{-\infty}^{\infty} x dx = \lim_{n \to \infty} 0 = 0$ (5 marks)
- b. Show the properties of a good measure (5 marks)
- c. Proof that every countable subset of R has outer measure 0. (5 marks)
- d. Suppose A and B are subsets of R with $A \subset B$ show that $|A| \le |B|$ (5 marks)
- e. Prove that if $A, B \subset R$ and $|A| \leq \infty$ then $|B \setminus A| \geq |B| |A|$. (10 marks)

SECTION B (20 MARKS)

QUESTION TWO

- a. Suppose *F* is a subset of *R* with the property that every open cover of *F* has a finite sub cover. Prove that *F* is closed and bounded.(10marks)
- b. Suppose \mathcal{T} is an σ -algebra on a setY and $X \in \mathcal{T}$. Let $S = \{E \in \mathcal{T}: E \subset X\}$. Show that Let $S = \{F \cap X: F \subset \mathcal{T}\}$ (10 marks)

QUESTION THREE

a. Suppose (X,S) is a measurable space and $f : X \to R$ is a function such that $f^{-1}((a,\infty))\mathcal{E}S$ for all $a\mathcal{E}R$. Then f is an S-measurable function.

(5 marks)

b. Define $f:[0,1] \rightarrow R$ by

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational,} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$$
(10 marks)

QUESTION FOUR

- a. Suppose X is a set and A is the set of subsets of X that consist of exactly one element: A = {{x}: x ∈ X}. Show the smallest σ −algebra. (10 marks)
- b. Prove that $L(f, P[a, b]) \le U(f, P'[a, b])$ if $f: [a, b] \to R$ (5 marks)
- c. Determine the number of roots of $z^7 4z^3 + z 1$ inside (5 marks)

QUESTION FIVE

- a. Define $f:[0,1] \rightarrow R$ by $f(x) = x^2$ by partitioning (5 marks)
- b. Show that every continuous real-valued function on each closed bounded interval is Riemann integrable (10 marks)
- c. Show that S is an σ algebra on X (5 marks)