KISII UNIVERSITY
UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF MASTERS IN PURE MATHEMATICS SECOND SEMESTER 2021/2022 (OCTOBER-JANUARY, 2022)

MATH 809: ABSTRACT INTEGRATION I

STREAM: : Y2S1 **TIME: 3 HOURS**

DAY: MONDAY, 8.00 AM – 11.00 AM **DATE: 01/03/2022**

INSTRUCTIONS:

1. ***Do not write anything on this question paper.***
2. ***Answer Question One (Compulsory) and Any Other Two Questions.***

SECTION A [30 MARKS]

QUESTION ONE

a. Show that $\mathrm{P.V}\int_{-\infty}^{\infty} x\,dx = \lim_{R\to\infty} 0 = 0$ (5 marks)

b. Show the properties of a good measure (5 marks)

c. Proof that every countable subset of R has outer measure 0. (5 marks)

d. Suppose A and B are subsets of R with $A \subset B$ show that $|A| \leq |B|$ (5 marks)

e. Prove that if $A, B \subset R$ and $|A| \leq \infty$then$|B\backslash A| \geq |B| - |A|$. (10 marks)

SECTION B (20 MARKS)

QUESTION TWO

a. Suppose F is a subset of R with the property that every open cover of F has a finite sub cover. Prove that F is closed and bounded.(10marks)

b. Suppose $\mathcal{T}$is an σ −algebra on a set$Y$and $X \in \mathcal{T}$. Let $S = \{E \in \mathcal{T}: E \subset X\}$. Show that Let $S = \{F \cap X: F \subset \mathcal{T}\}$ (10 marks)

QUESTION THREE

a. Suppose (X, S) is a measurable space and $f : X \to R$ is a function such that $f^{-1}((a, \infty)) \mathcal{E} S$for all $a \mathcal{E} R$. Then f is an$S$ −measurable function. (5 marks)

b. Define $f: [0,1] \to R$ by

$$f(x) = \begin{cases} 1 & \text{if x is rational,} \\ 0 & \text{if x is irrational.} \end{cases}$$ (10 marks)

QUESTION FOUR

a. Suppose X is a set and A is the set of subsets of X that consist of exactly one element: $A = \{\{x\}: x \in X\}$. Show the smallest σ −algebra. (10 marks)

b. Prove that $L(f, P[a, b]) \leq U(f, P'[a, b])$ if $f: [a, b] \to R$ (5 marks)

c. Determine the number of roots of $z^7 - 4z^3 + z - 1$ inside (5 marks)

QUESTION FIVE

a. Define $f: [0,1] \to R$by $f(x) = x^2$by partitioning (5 marks)

b. Show that every continuous real-valued function on each closed bounded interval is Riemann integrable (10 marks)

c. Show that S is an $\sigma -$ algebra on X (5 marks)