KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF MASTERS IN PURE MATHEMATICS
SECOND SEMESTER 2021/2022
(FEBRUARY-MARCH, 2022)

MATH 812: COMPLEX ANALYSIS II

STREAM: : Y1S2 **TIME: 3 HOURS**

DAY: MONDAY, 8.00 AM – 11.00 AM **DATE: 26/02/2022**

INSTRUCTIONS:

1. ***Do not write anything on this question paper.***
2. ***Answer Question One (Compulsory) and Any Other Two Questions.***

QUESTION ONE

SECTION A [30 MARKS]

1.

a. Show that $\mathrm{P.V}\int_{-\infty}^{\infty} x\,dx = \lim_{R\to\infty} 0 = 0$ (5 marks)

b. Use the function $f(z) = \frac{z^2}{z^6+1}$ to evaluate the integral $\int_0^{\infty} \frac{z^2}{z^6+1} dx$(5marks)

c. Show that $\int_{-\infty}^{\infty} \frac{\cos 3x}{(x^2+1)^2} dx = \frac{2\pi}{e^3}$ (5 marks)

d. State Jordan's Lemma (5 marks)

e. Find the Cauchy principal value of $\int_{-\infty}^{\infty} \frac{x\sin x\,dx}{x^2+2x+2}$ (5 marks)

f. Show that $\int_{-\infty}^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$ (5 marks)

SECTION B (20 MARKS)

i. Suppose the points $z_1 = 1, z_2 = 0, z_3 = -1$ are mapped onto $w_1 = i, w_2 = \infty, w_3 = 1$, show the type of transformation used(5 marks)

ii. Show that $y = c_2$is mapped by $w = \frac{1}{z}$ onto a circle (5 marks)

iii. Find the Laurent series for $f(z) = \frac{1}{(z-i)^2}$ at $z = i$ (5 marks)

iv. Compute $\int_0^{1+i} z^2 dz$ (5 marks)

QUESTION TWO

i. State and proof Scharz-Christoffel theorem of transformation (10marks)

i. Locate the vertices of a rectangle $a > 1$ where $x_1 = -a$, $x_2 = -1$, $x_3 = 1$ and $x_4 = a$ (10 marks)

QUESTION THREE

i. Find the function $f(t)$ that corresponds to $F(s) = \frac{s}{(s^2+a^2)^2}$ $(a > 0)$

(10 marks)

ii. Show that mapping $w = (1 + i)z + 2$ transforms the rectangular region in the $z = (x, y)$into a rectangular region $w = (u, v)$with inclination angle$\frac{\pi}{4}$ (5 marks)

iii. Find the special case of transformation $z_1 = -1, z_2 = 0, z_3 = 1$ onto points $w_1 = -i, w_2 = 1, w_3 = 1$ (5 marks)

QUESTION FOUR

a. Determine the number of roots of $z^7 - 4z^3 + z - 1$ inside

a circle $|z| = 1$ (5 marks)

b. Show that $\int_0^{2\pi} \frac{d\theta}{1+a\sin\theta} dx = \frac{2\pi}{\sqrt{1-a^2}}$ (5 marks)

c. State and proof Rouche's Theorem (10 marks)