KISII UNIVERSITY

FIRST YEAR SEMESTER TWO EXAMINATION FOR THE

DEGREE OF MASTER OF SCIENCE- PURE MATHEMATICS

MATH 802- FUNCTIONAL ANALYSIS II

TIME- 3 HOURS

STREAM: MSC PURE MATHEMATICS

INSTRUCTIONS: Answer question one (compulsory) and any other two questions

QUESTION ONE (30 MARKS)- COMPULSORY

a) i) Define an inner product (2mks)

ii) Show that the space $\mathbb{R}^n$ with $\langle , \rangle$defined by $\langle x, y\rangle = \sum_{i=1}^{n} x_i y_i$ for $x, y \in \mathbb{R}^n$is an inner product space (8mks)

iii) State and proof the Cauchy-Scwartz inequality (8mks)

b) i) Define a Hilbert space (1mk)

ii) Let X be an inner product space and $x, y \in X$. If $x \perp y$, prove that $\|x+y\|^2 = \|x\|^2 + \|y\|^2$ (5mks)

c) State and proof the Riesz representation theorem (6mks)

QUESTION TWO (20 MARKS)

a) Define the term an adjoint operator of a Hilbert space (2mks)

b) Let T be a bounded linear operator on a Hilbert space H onto itself, prove that

i) $\|T\| = \|T^*\|$ (5mks)

ii) $\|T^*T\| \leq \|T\|^2$ (5mks)

c) i) What is a self-adjoint operator? (1mk)

ii) Let $T : l_2\left(0, 1\right) \rightarrow l_2\left(0, 1\right)$ be defined by $Tf\left(t\right) = tf\left(t\right)$. Show that T is self-adjoint (7mks)

QUESTION THREE (20 MARKS)

a) i) Define a normal operator (1mk)

ii) Let $T : H \rightarrow H$ be a bounded linear operator on H, Proof that T is normal iff $\|T^*x\| = \|Tx\| \; \forall \; x \in X$ (7mks)

c) Let T be an operator on a Hilbert space, show that the following statements are equivalent

i) $T^*T = I$ (4mks)

ii) $\langle Tx, Ty\rangle = \langle x, y\rangle$ (4mks)

iii) $\|Tx\| = \|x\|$ (4mks)

QUESTION FOUR (20 MARKS)

a)i) Define a reflexive space (1mk)

ii) Prove that every finite dimensional normed space X is reflexive (5mks)

b) Prove that every Hilbert space is reflexive (5mks)

c) i) State the Milman-pettis theorem (1mk)

ii) Define the spectrum of a linear operator (1mk)

iii) Prove that every compact operator is bounded (7mks)