KISII UNIVERSITY

FIRST YEAR SEMESTER TWO EXAMINATION FOR THE

DEGREE OF MASTER OF SCIENCE- PURE MATHEMATICS

MATH 804- GENERAL TOPOLOGY II

TIME- 3 HOURS

STREAM: MSC PURE MATHEMATICS

INSTRUCTIONS: Answer question one (compulsory) and any other two questions

QUESTION ONE (30 MARKS)- COMPULSORY

a) i) Define an open cover of a topological space (1 mk)

ii) State the Heine -Borel theorem (1mk)

iii) Define compactness in a topological set (1mk)

iv) Let A be any finite subset of a topological space X, prove that A is necessarily compact (4mks)

b) i) Define the term finite intersection property (2mks)

ii) Prove that every bijective continuous function from a compact space onto a Hausdorff space is a homeomorphism (7mks)

c) i) Define local compactness (2mks)

ii) State without proof the stone- cech theorem (2mks)

d) i) What is a disconnected space (2mks)

ii) Show that if A and B are non-empty separated sets, then $A \cup B$ is disconnected (8mks)

QUESTION TWO (20 MARKS)

a) Define local connectedness (2mks)

b) Let E be a component in a locally connected space X, prove that E is open (6mks)

c) i) Define the tychonoff topology (2mks)

ii) State the tychonoff's theorem (3mks)

d) Let $\{X_i : i \in I\}$be a collection of Hausdorff spaces and let X be be the product space. Show that X is also a Hausdorff space (7mks)

QUESTION THREE (20 MARKS)

a) i) Define a covering space (2mks)

ii) If $f : E \to B$ and $f' : E' \to B'$ are covering maps, prove that $f \times f' : E \times E' \to B \times B'$ is a covering map (8mks)

b) i)Define the identification topology (2mk)

ii) Prove that if $p : X \to Y$ is a continuous open surjection, then it is an identification topology (8mks)

QUESTION FOUR (20 MARKS)

a) Prove that continuous images of compact sets are compact (10mks)

b) Show that every closed subset of a compact space X is compact (10mks)