KISII UNIVERSITY

FIRST YEAR SEMESTER TWO EXAMINATION FOR THE

DEGREE OF MASTER OF SCIENCE- PURE MATHEMATICS

MATH 806- GROUP THEORY II

TIME- 3 HOURS

STREAM: MSC PURE MATHEMATICS

INSTRUCTIONS: Answer question one and any other two questions

<u>QUESTION ONE (30 MARKS)- COMPULSORY</u>

a) i) Define the term a representation of a group G (2mks)

ii) Let $G \leq S_n$ for some n. Let $V = \mathbb{R}$and $F = \mathbb{R}$.

Define$T(x) = 1$, if x is even and $T(x) = -1$, if x is odd. Show that T is a linear representation of G. (8mks)

b) Let $G = S_2 = \{g_1 = 1, g_2 = (1,2)\}$ and V be a two- dimensional vector space with basis $\{v_1, v_2\}$. Find the regular representation of G(5mks)

c) State and proof Maschkes theorem (8mks)

d) i) Define the term T-invariance of a representation of a group (1mk)

ii) Suppose that $\{W_i\}$ is a collection of T-invariant subspaces of a vector space V, show that the intersection $W = \cap W_i$ is also T-invariant. (6mks)

QUESTION TWO (20 MARKS)

a) i) What is meant by a character of a group G (1mk)

ii) If χ and θ are F-characters. Prove that

$$(\chi,\theta)=\sum_{i=1}^{k}(\chi,\chi_i)(\theta,\chi_i)$$ (5mks)

b)i) Distinguish between a decomposable and indecomposable representation of a group (2mks)

ii) Let $G=S_3$ and $S=\{1,2,3\}$. Find the permutation representation as matrices for S_3 (12mks)

QUESTION THREE (20 MARKS)

a) i) Suppose char F=0 and χ is an F-Character of a group G, prove that χ is irreducible if and only if $(\chi,\chi)=1$ (5mks)

ii) Prove that characters are class functions (5mks)

b)i) State without proof the orbit-stabilizer theorem (1mk)

ii) If G acts on a set $A=\{a_1,a_2,......,a_k\}$ with corresponding T having character θ. Show that (θ,I_G) =number of orbits of G on A=1 (9mks)

QUESTION FOUR (20 MARKS)

a) Compute the character table for the quarternion group (8mks)

b) Let T be a representation of a group G on a vector space V. Show that each of the following is T-invariant

i) $\{\underset{\sim}{0}\}$ (3mks)

ii) V (3mks)

iii) Ker of T (3mks)

iv) Image of T (3mks)