KISII UNIVERSITY

EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN PURE MATHEMATICS

MATH 822- OPERATOR THEORY II

STREAM- MSC Y2S2

TIME: 3 HOURS

INSTRUCTIONS: ANSWER QUESTION ONE COMPULSORY AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) i) Define a Banach algebra (1mk)

ii) Let A be a unital Banach algebra and $\lambda \in A$ such that $\|\lambda\| < 1$.

Prove that $1 - \lambda$ is invertible and $\| (1 - \lambda)^{-1} \| \leq \frac{1}{1-\|\lambda\|}$ (10mks)

b) Let X be an element of a Banach algebra A and $|\lambda| > \|x\|$, prove

that $(\lambda I - x)^{-1} = \sum_{n=0}^{\infty} \frac{x^n}{\lambda^{n+1}}$ (10mks)

c) i) What is a homomorphism? (1mk)

ii) Let X be a Banach algebra and $f : X \longrightarrow \mathbb{C}$ be a homomorphism.

Prove that $|f(x)| \leq \|x\| \forall x \in X$ (7mks)

iii) State without proof the Gelfand- Mazur theorem (2mks)

QUESTION TWO (20 MARKS)

a) i) Distinguish between an involution and a $C*-$algebra (4mks)

ii) If a is a self adjoint element of a $C*-$algebra A, prove that

$r(a) = \|a\|$ (4mks)

b) i) Define a symmetric linear operator (1mk)

ii) Prove that a densely defined linear operator T in a complex Hilbert

space H is symmetric iff $T \subset T^*$ (8mks)

c) Prove that a self-adjoint linear operator T is maximally symmetric (3mks)

QUESTION THREE (20 MARKS)

a) i)Define a closed linear operator (2mks)

ii) Prove that the Hilbert adjoint operator T^*defined by $T^*y = y*$ is closed (8mks)

b) Prove that every non-expansive mapping is pseudo-contractive (10mks)

QUESTION FOUR (20 MARKS)

a) i) Define a partial isometry (1mk)

ii) Let U be a a partial isometry in H, prove that $U * U$ is an orthogonal projection (9mks)

b) i) Define the cayley transform (1mk)

ii) Prove that if the cayley transform of a symmetric operator is unitary, then the operator is self adjoint (9mks)