KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF
THE DEGREE OF MASTER OF SCIENCE APPLIED STATISTICS
FIRST SEMESTER 2022/2023
(JUNE-SEPTEMBER, 2022)

MATH 873: MULTIVARIATE ANALYISI

STREAM: Y1S1 **TIME: 3 HOURS**

DAY: FRIDAY, 8.00 AM – 11.00 AM **DATE: 23/09/2022**

INSTRUCTIONS:

1. ***Do not write anything on this question paper.***
2. ***Answer Question ONE (Compulsory) and Any Other TWO Questions***

QUESTION ONE (30 MARKS)

1 a) State three objectives of Multivariate data analysis (3marks)

b) Explain the meaning of correlation coefficient in multivariate analysis (2marks)

c) Show that Var(X) can be written as $\sum = E[XX^T] - \mu\mu^T$ (4marks)

d) Given that $\sum = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 9 & -3 \\ 2 & -3 & 25 \end{bmatrix}$

i) Obtain $V^{\frac{1}{2}}$ (2marks)

ii) ρ (2marks)

e) Given that $X \sim N_3(\mu, \varepsilon)$where $\sum = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, determine whether

i) X_1 and $X_2$are independent (1mark)

ii) $(X_1 X_2)$ and $X_3$are independent (3marks)

f) Let a $\neq 0$ where $\underset{\sim}{a}$ is a $p \times 1$ vector of constant. Show that $Var(a^T X) = a^T \sum a$ (5marks)

g) Consider a p-variate normal distribution $f(x) = \left(\frac{1}{2\pi}\right)^{\frac{p}{2}} \frac{1}{|\Sigma|^{\frac{1}{2}}} exp^{-\frac{1}{2}Q}$

where $Q = \left(\underset{\sim}{X} - \underset{\sim}{\mu}\right)^1 \Sigma^{-1} \left(\underset{\sim}{X} - \underset{\sim}{\mu}\right)$. Show that

i) $\underset{\sim}{X} = \underset{\sim}{\mu}$ (5marks)

ii) $Q = \underset{\sim}{X}^1 \Sigma^{-1} \underset{\sim}{X} - \underset{\sim}{X}^1 \Sigma^{-1} \underset{\sim}{\mu} - \underset{\sim}{\mu}^1 \Sigma^{-1} \underset{\sim}{X} + \underset{\sim}{\mu}^1 \Sigma^{-1} \underset{\sim}{\mu}$ (3marks)

QUESTION TWO (15 MARKS)

a) Explain HotellingsT^2statistics (1mark)

b) Outline the procedure of testing hypothesis using HotellingsT^2statistics (4marks)

c) let a matrix for a random sample of size $n = 7$ from a bivariate normal population be

$X = \begin{Bmatrix} 15 & 13 & 12 & 15 & 17 & 10 & 16 \\ 19 & 15 & 17 & 21 & 24 & 20 & 17 \end{Bmatrix}$. Test the hypothesis that; $H_0: \mu = [17 \;\; 15]^1$ vs $H_1: \mu \neq [17 \;\; 15]^1$ at α=0.05 (10marks)

QUESTION THREE (15 MARKS)

2 a) Write down the bivariate normal distribution (3marks)

b) Given that $X \sim N_3(\mu, \varepsilon)$ find the distribution of $Y = \begin{bmatrix} X_1 - X_2 \\ X_2 - X_3 \end{bmatrix}$ (6marks)

c) Suppose $X \sim N_5(\mu, \varepsilon)$ find the distribution of $\begin{bmatrix} X_2 \\ X_4 \end{bmatrix}$ (6marks)

QUESTION FOUR (15 MARKS)

a) Given a bivariate random vector $\underset{\sim}{X} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, explain conditional distribution

b) of x_2 given x_1 (2marks)

c) Given $\underset{\sim}{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$, $\underset{\sim}{\mu} = \begin{pmatrix} 2 \\ 5 \\ -2 \\ 1 \end{pmatrix}$, $\Sigma = \begin{bmatrix} 9 & 0 & 3 & 3 \\ 0 & 1 & -1 & 2 \\ 3 & -1 & 6 & -3 \\ 3 & 2 & -3 & 7 \end{bmatrix}$ and that

$\underset{\sim}{Y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$, $\underset{\sim}{V} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

Obtain

i) $E\left(\underset{\smile}{Y} \mid \underset{\smile}{V}\right)$ (4marks)

ii) $Var\ \left(\underset{\smile}{Y} \mid \underset{\smile}{V}\right)$ (4marks)

d) Suppose $U = a'X$ and $V = b'X$, show that $COV(U,V) = a' \sum b$ (5marks)

QUESTION FIVE (15 MARKS)

a) Suppose X is a random vector, show that $var(X) = E[XX^T] - \mu\mu^T$ (4marks)

b) Given that $U = a^T X \quad and \quad V = b^T X$ where X is a random vector and$b$ and a are some vectors of constants. Determine $Cov(U,V)$ (5marks)

c) Given that a p-variate normal distribution i.e$\underset{\smile}{X} \sim N_p(\mu, \Sigma)$.

d) Define $Y = \Sigma^{-\frac{1}{2}}\left(\underset{\smile}{X} - \underset{\smile}{\mu}\right)$ find the density function of $Y\ i.e\ f(y)$(6marks)