KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF MASTER OF SCIENCE APPLIED STATISTICS

FIRST SEMESTER 2022/2023

(JUNE-SEPTEMBER, 2022)

MATH 882: MEASURE THEORY AND PROBABILITY

STREAM: Y1S1 **TIME: 3 HOURS**

DAY: THURSDAY, 2.00 PM – 5.00 PM **DATE: 22/09/2022**

INSTRUCTIONS:

1. ***Do not write anything on this question paper.***
2. ***Answer Question ONE (Compulsory) and Any Other THREE Questions***

QUESTION ONE-25 MARKS

a) Define
   i) Lebesgue-Stieltjes Measure on R. [5 Marks]
   ii) Distribution Function [5 Marks]
   iii) Measurable Space [5 Marks]

b) Let v be a measurable function on a measure space (X, A). Define the terms below;

i) Positive set with respect to v. [5 Marks]
ii) Negative set with respect to v. [5 Marks]

QUESTION TWO-15 MARKS

Let (X, A, μ) be a measure space and let s be a non negative measurable simple function on X. Define φ and A by;

$$\varphi(E) = \int_E s d\mu \quad (E\epsilon A$$

Then φ is a measure on (X,A). Proof.

QUESTION THREE-15 MARKS

Let f be a measurable function on a measure space (X, A, μ). Then f is integrable iff $|f|$ is integrable. Proof

QUESTION FOUR-15 MARKS

Let f be a measurable function on a measure space (X, A, μ) such that $\int_X f d\mu$ exists. Then $\left|\int_X f d\mu\right| \leq \left|\int_X f d\mu\right|$. Proof.

QUESTION FIVE-15 MARKS

State and proof Monotonicity of a Measure.