KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE MATHEMATICS AND COMPUTING
SECOND SEMESTER 2022/2023
[JUNE - SEPTEMBER, 2022]

MATH 206: INTRODUCTION TO ANALYSIS

STREAM: Y2 S2 **TIME: 2 HOURS**

DAY: THURSDAY, 3.00 PM – 5.00 PM **DATE: 08/09/2022**

INSTRUCTIONS:

1. Do not write anything on this question paper.
2. Answer question ONE [Compulsory] and any other TWO Questions.

QUESTION ONE

a) Define a rational number [2 marks]

b) Show that between any two rational numbers there exists another rational number. [4 marks]

c) Show that $\sqrt{5}$ is not an irrational number [5 marks]

d) Distinguish between odd and even numbers [2 marks]

e) Let $m \in \mathbb{Z}$, prove that$m$ is even iff m^2 is even [5 marks]

f) Prove that if x and y are negative numbers, then $x + y$ is also negative [4 marks]

g) If $\forall x, \in \mathbb{R}$, show that$x < y$ iff $x^2 < y^2$[5 marks]

a) Determine whether the set $S = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$ is bounded or not 3mks

QUESTION TWO

b) Find the supremum and infimum of the set $S = \{1 + (-1)^n : n \in \mathbb{N}\}$ [5 marks]

c) Let S be a non-empty subset of $\mathbb{R}$. Prove that the real number a is the infimum of the set S if and only if the following conditions are satisfied

i) $a \leq x, \forall \; x \in S$

ii) $\forall \epsilon > 0, \exists \, x' \in S : \; a \leq x < x' < a + \epsilon$ [6 marks]

d) State the completeness axiom for real numbers [2 marks]

e) Prove tha if N_1 and $N_2$are neighborhoods of the point x, then $N_1 \cap N_2$ is also a neighborhood of x [4 marks]

f) Determine whether the given subset of $\mathbb{R}$ is open or closed $S = \{(-1)^n : n \in \mathbb{N}\}$ [3 marks]

QUESTION THREE

a) Show that the set $T = (-2,\ 2)$ is open [4 marks]

b) Prove that the finite intersection of open sets is also open [5 marks]

c) Show that $\lim_{x \to \infty} -6x + 3 = 15$ from first principles [4 marks]

d) State and prove the Darboux's integrability condition [7marks]

QUESTION FOUR

a) When do we say that a sequence is convergent? [1mark]

b) From first principle, show whether a sequence $X_n = \frac{n}{n+}$ is convergent or not [4 marks]

c) Show that the limit of a convergent sequence is unique [3 marks]

d) Prove that every convergent sequence is bounded [4 marks]

e) Find the limit superior and the limit inferior of the sequence $X_n = \left\{\cos\frac{n\pi^{C}}{2} + \frac{1}{n}\sin\left(\frac{2n+1}{2}\right)\pi\right\}$ [8 marks]

QUESTION FIVE

a) Define a continuous function [2 marks]

b) Show that a function $f(n) = \frac{n^2-1}{n^2+1}$ converges to 1 as $n \to \infty$ [4 marks]

c) Prove that if a limit of a function exists, then the limit is unique. [3 marks]

d) Show that the set $\mathbb{R}$ is not countable[5 marks]

e) Show that a constant function $f(x) = K$ on $[a, b\ ]$ is Riemann integrable and find its integral. [6 marks]